Fundamentals of Solid State Physics

Semiconductors - General

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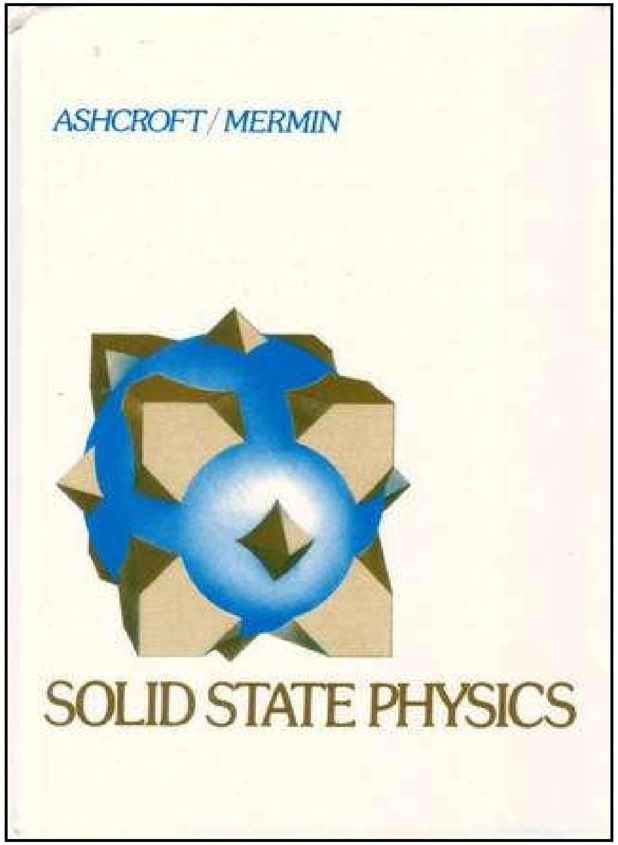
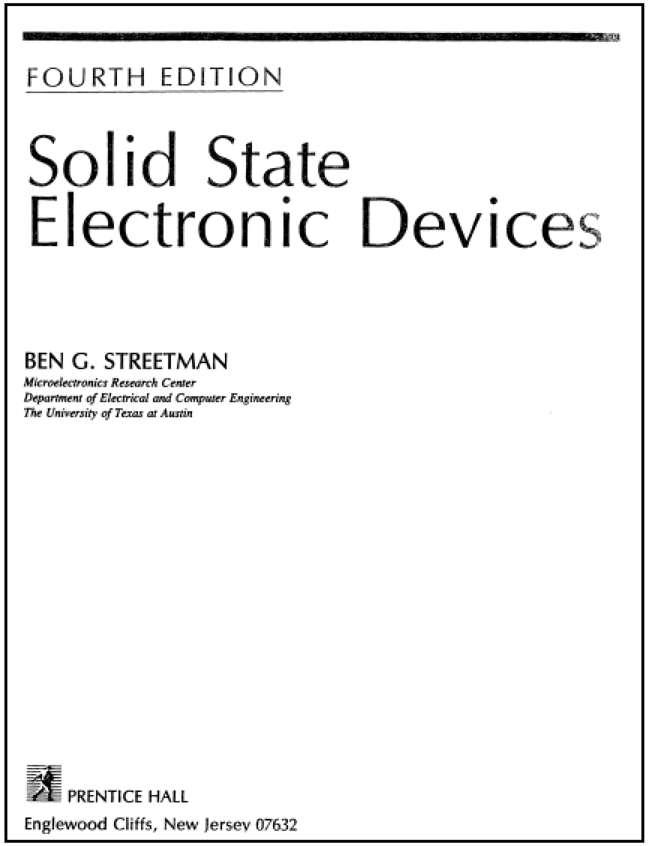
Further Reading

- **Ashcroft & Mermin, Chapter 28**
- **Solid State Electronic Devices by Streetman, Chap.3**



ASHCROFT / MERMIN

SOLID STATE PHYSICS



FOURTH EDITION

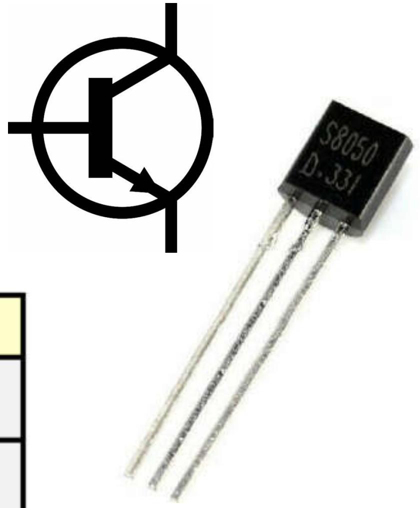
Solid State
Electronic Devices

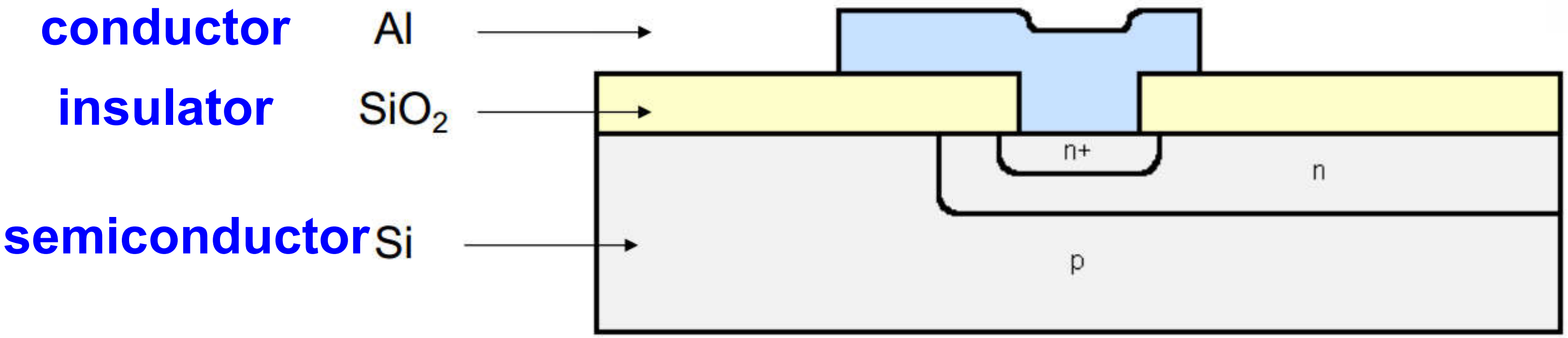
BEN G. STREETMAN
Microelectronics Research Center
Department of Electrical and Computer Engineering
The University of Texas at Austin

PRENTICE HALL
Englewood Cliffs, New Jersey 07632

Electronic Properties of Materials

CMOS transistor
- Complementary Metal-Oxide-Semiconductor



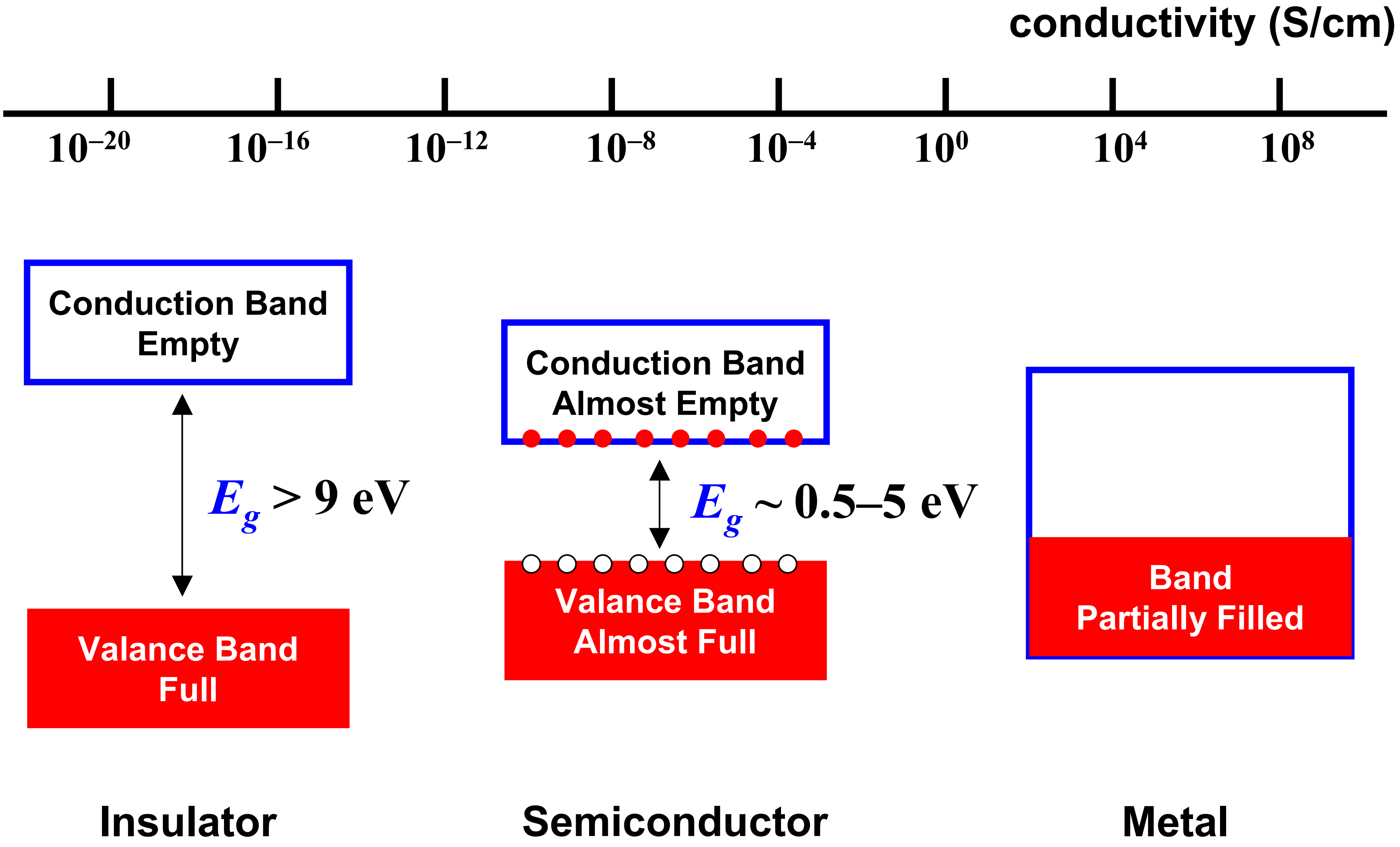


Metal

SiO_2

Silicon

Metal, Insulator, Semiconductor



conductivity (S/cm)

10^{-20} 10^{-16} 10^{-12} 10^{-8} 10^{-4} 10^{0} 10^{4} 10^{8}

Conduction Band
Empty

$E_g > 9$ eV

Valance Band
Full

Conduction Band
Almost Empty

$E_g \sim 0.5–5$ eV

Valance Band
Almost Full

Band
Partially Filled

Insulator

Semiconductor

Metal

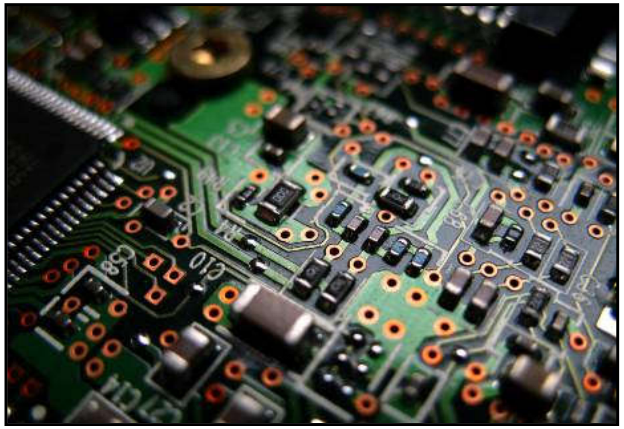
Semiconductors - General Concepts

- Band diagram $E(k)$
- Band gap E_g
- Effective mass m^*
- Holes
- Density of States (DOS) $g(E)$
- Density of Carriers n_c and p_v
  - Mass Action Law

- Intrinsic and Extrinsic

Semiconductors - Applications

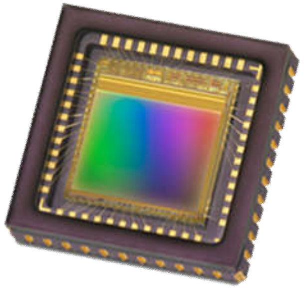
semiconductors are the basis of electronics and photonics



integrated circuits

LEDs

lasers



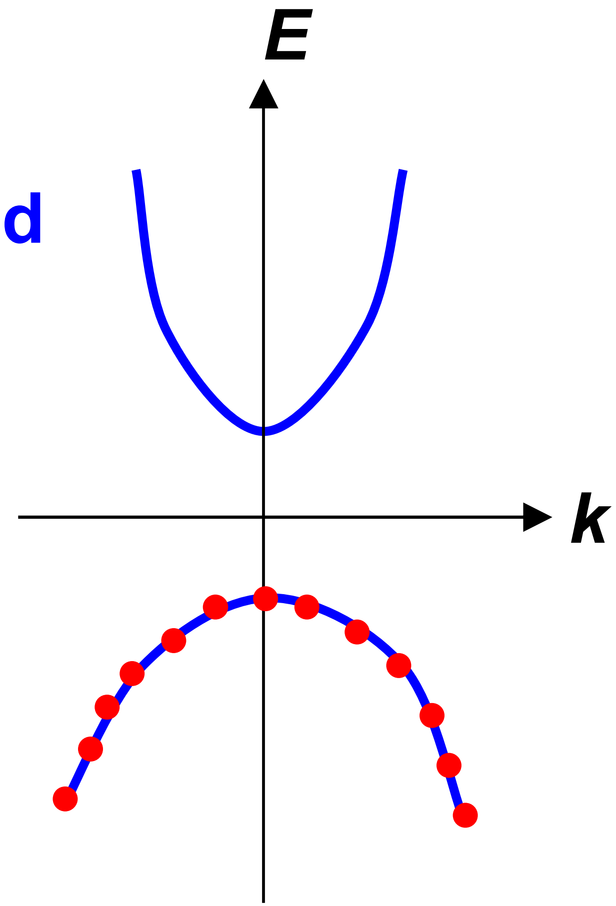
detectors

solar cells

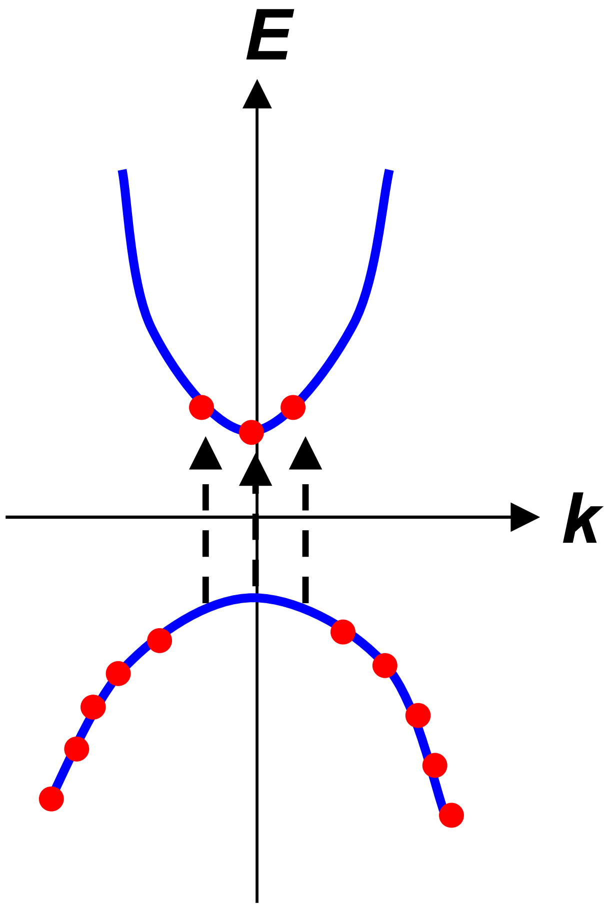
Semiconductor 半导体

conduction band
(CB) 导带

valence band
(VB) 价带



***T* = 0 K**
CB is empty, VB is full
insulator



***T* > 0 K**
thermalization 热激发
CB and VB are partly filled
conductor

Band Structure / Diagram 能带图

Free electrons

energy

$$E(k) = \frac{\hbar^2 k^2}{2m}$$

velocity

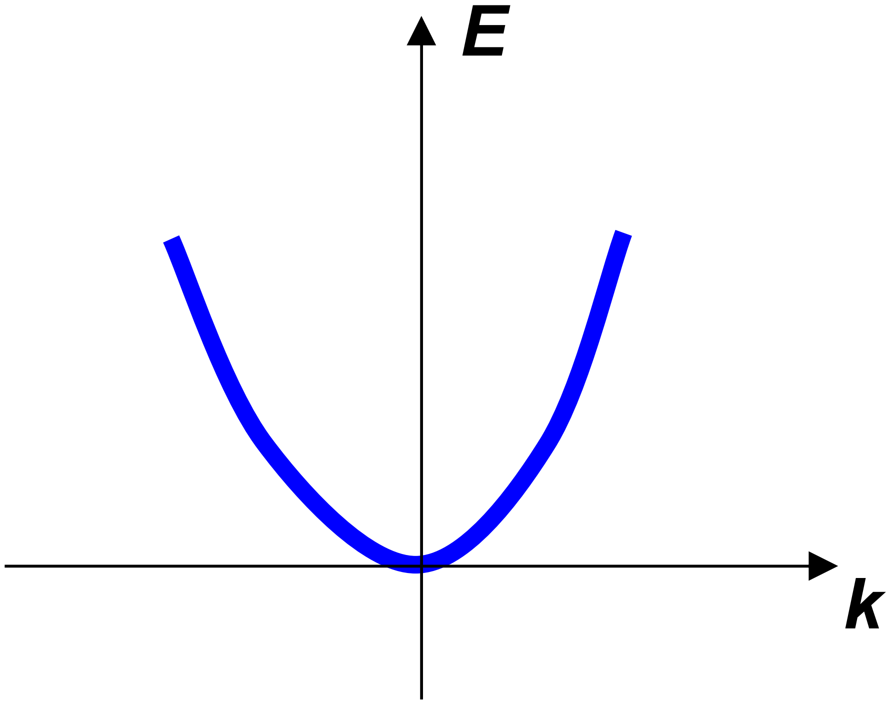
$$v = \frac{\hbar k}{m} = \frac{1}{\hbar}\frac{dE(k)}{dk}$$

momentum

$$p = \hbar k$$

electron mass

$$\frac{1}{m} = \frac{1}{\hbar^2}\frac{d^2E(k)}{dk^2}$$



***E*-*k* diagram**
(energy dispersion curve)

Band Structure / Diagram 能带图

energy $\boxed{E(k)}$

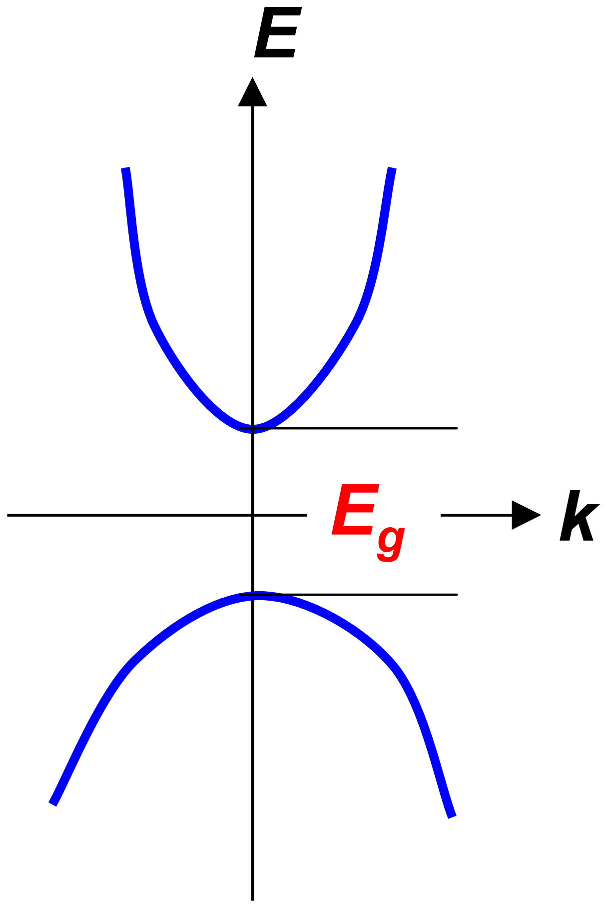
band gap $\boxed{E_g}$

crystal momentum
(*not* electron momentum) $\boxed{\hbar k}$

group velocity $\boxed{v_g = \frac{1}{\hbar}\frac{dE(k)}{dk}}$

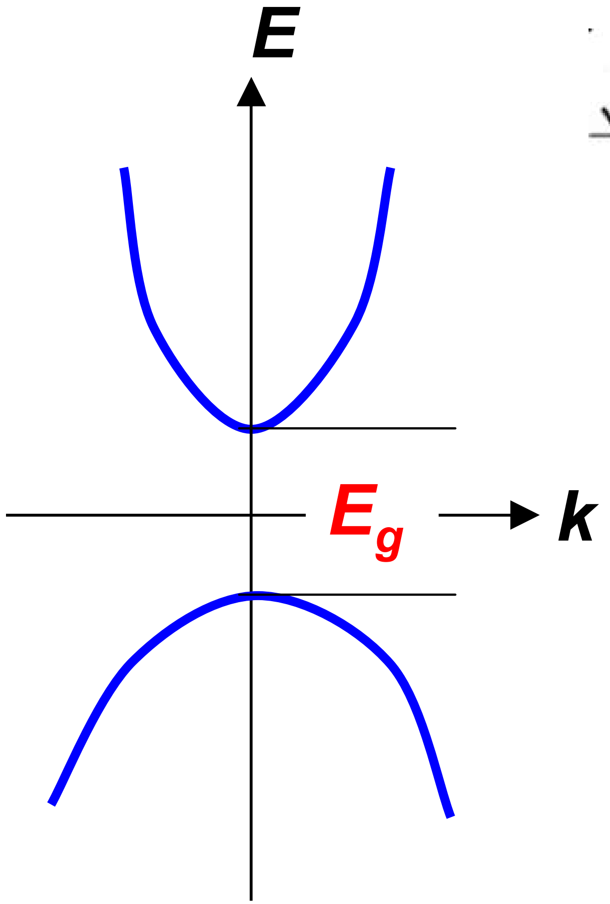
or

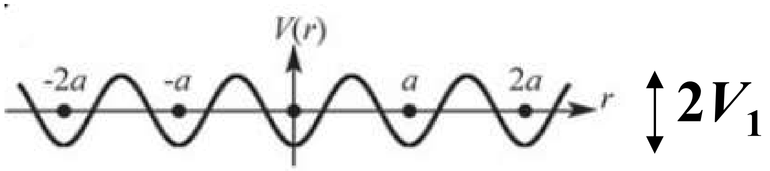
$$\boxed{\mathbf{v} = \frac{1}{\hbar}\nabla_{\mathbf{k}} E(\mathbf{k})}$$



***E-k* diagram**
(energy dispersion curve)

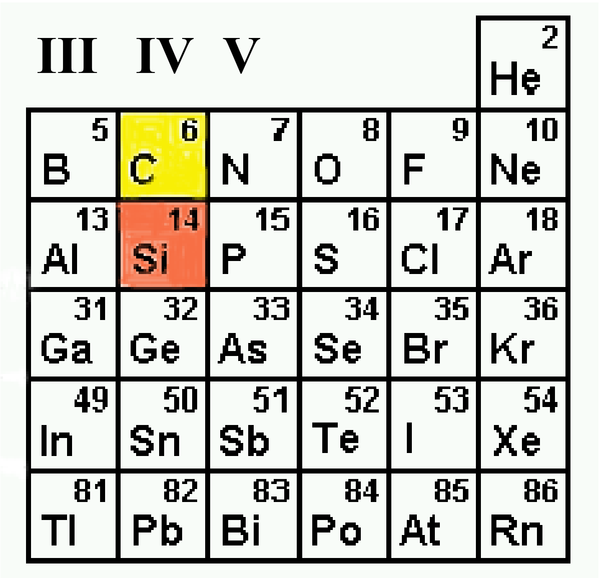
Band Gap E_g





$$\boxed{E_g = 2V_1}$$

the nearly free electron model



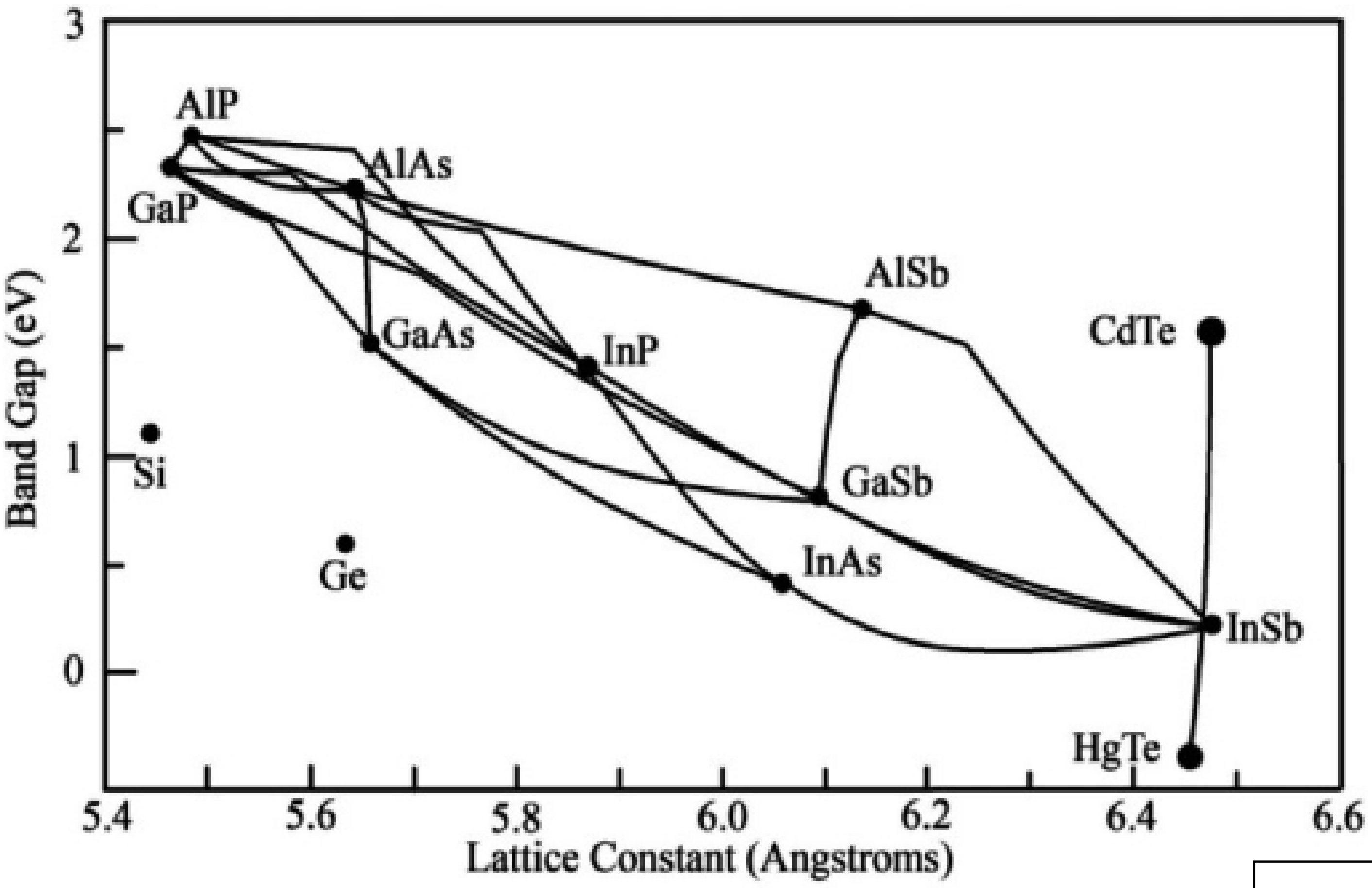
III	IV	V			2 He
5 B	6 C	7 N	8 O	9 F	10 Ne
13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn

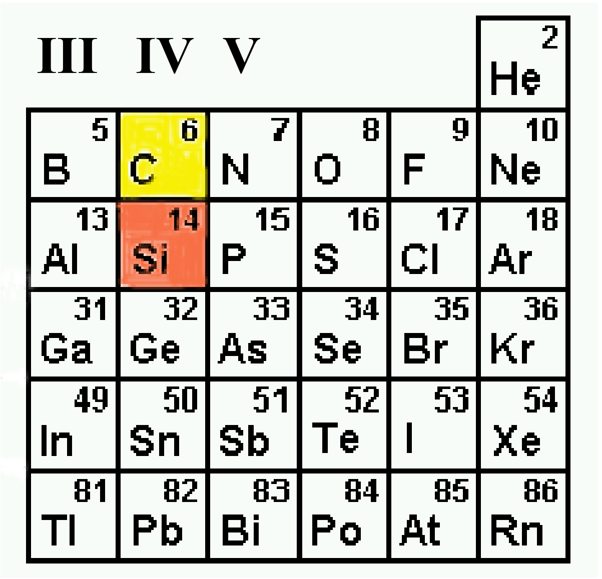
at T = 300 K

	a (Å)	E_g (eV)
C (diamond)	3.57	5.5
Si	5.43	1.1
Ge	5.66	0.66
α-Sn	6.49	0.08

Q: Why?

Band Gap E_g





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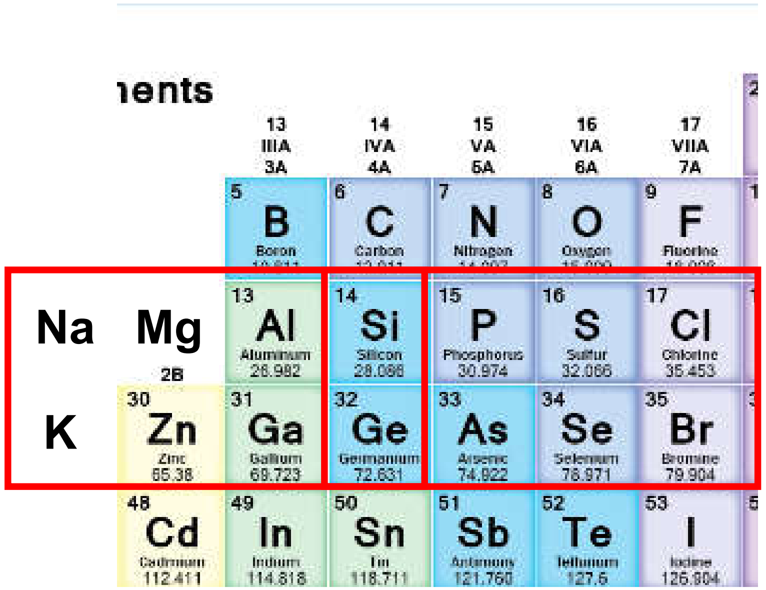
Si > Ge
AlAs > GaAs > InAs
GaP > GaAs > GaSb

larger atoms
-> smaller V_1
-> smaller E_g

Band Gap E_g

	E_g (eV)
Si	1.1
AlP	2.5
MgS	4.0
NaCl	8.5

	E_g (eV)
Ge	0.7
GaAs	1.4
ZnSe	3.5
KBr	7.5

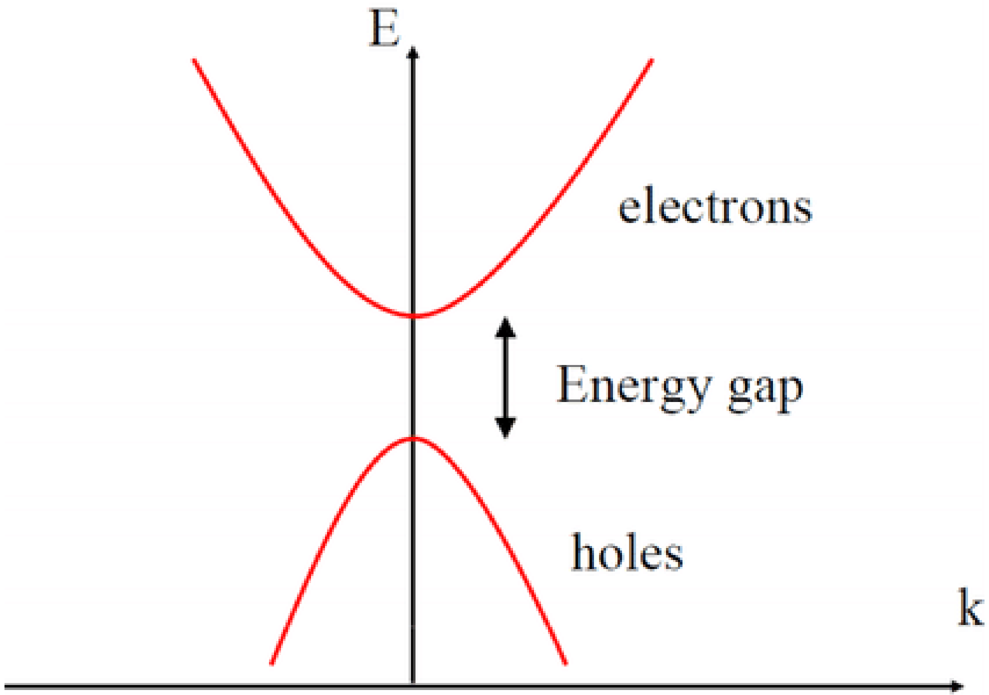


Si < AlP < MgS < NaCl

Ge < GaAs < ZnSe < KBr

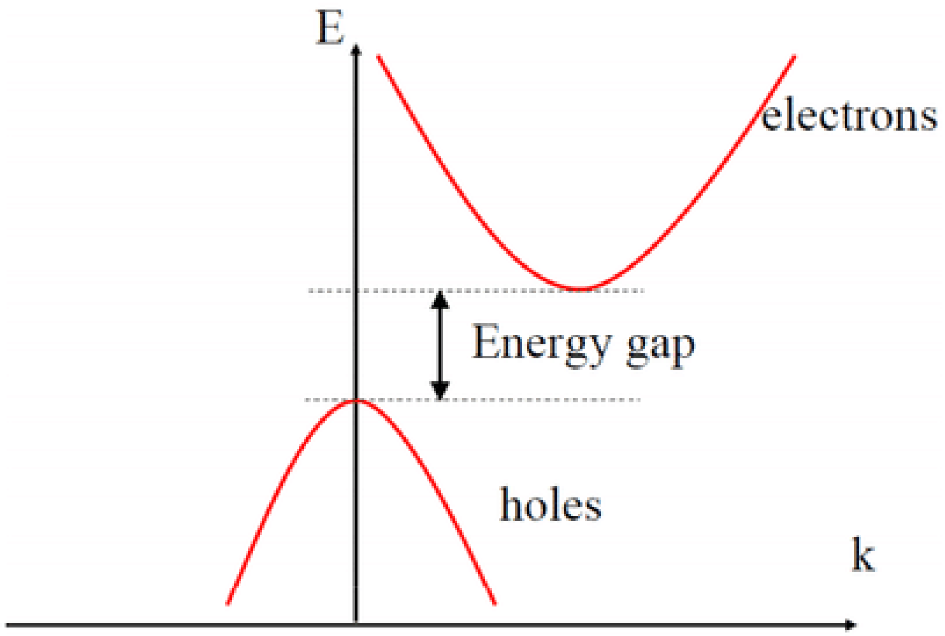
more polarity
-> larger V_1
-> larger E_g

Direct and Indirect Gaps



Direct bandgap semiconductors

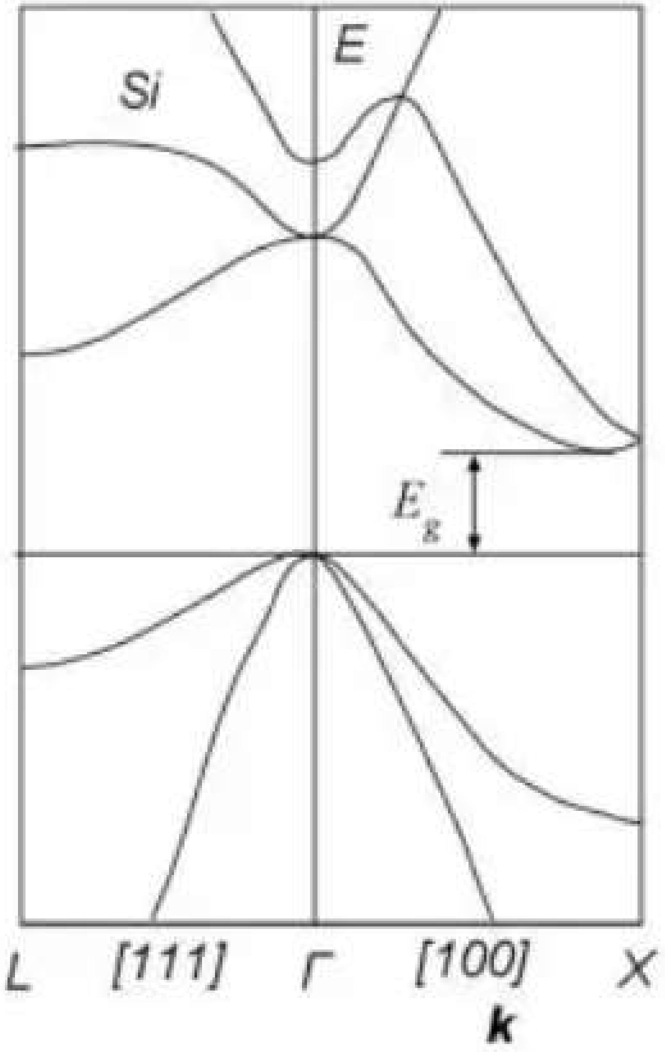
直接带隙



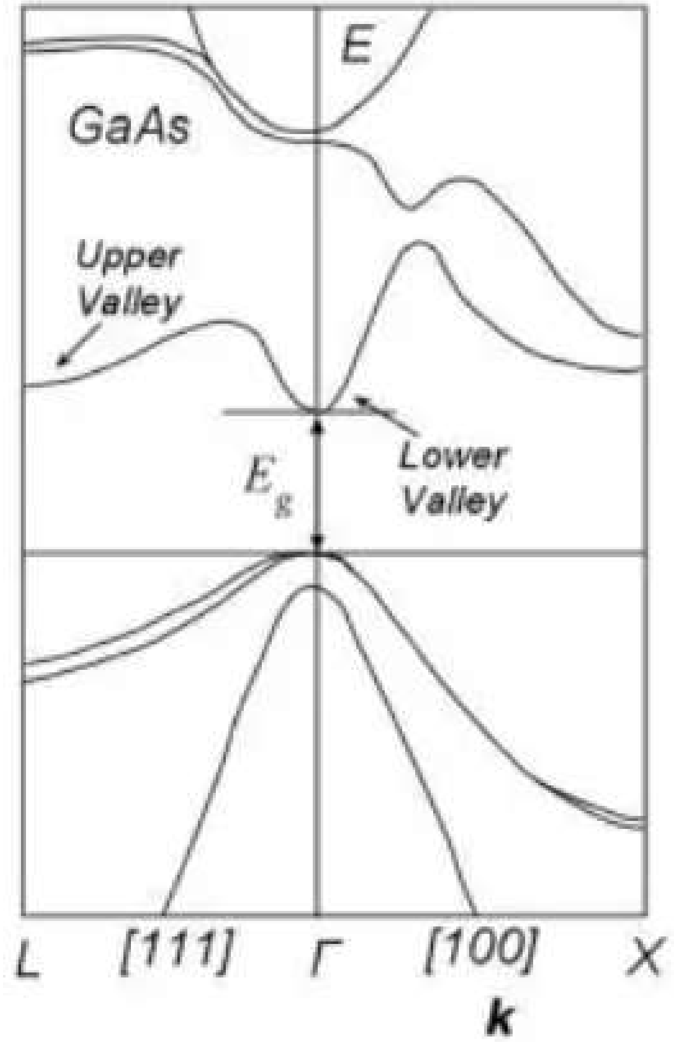
Indirect bandgap semiconductors

间接带隙

Direct and Indirect Gaps



Silicon - indirect



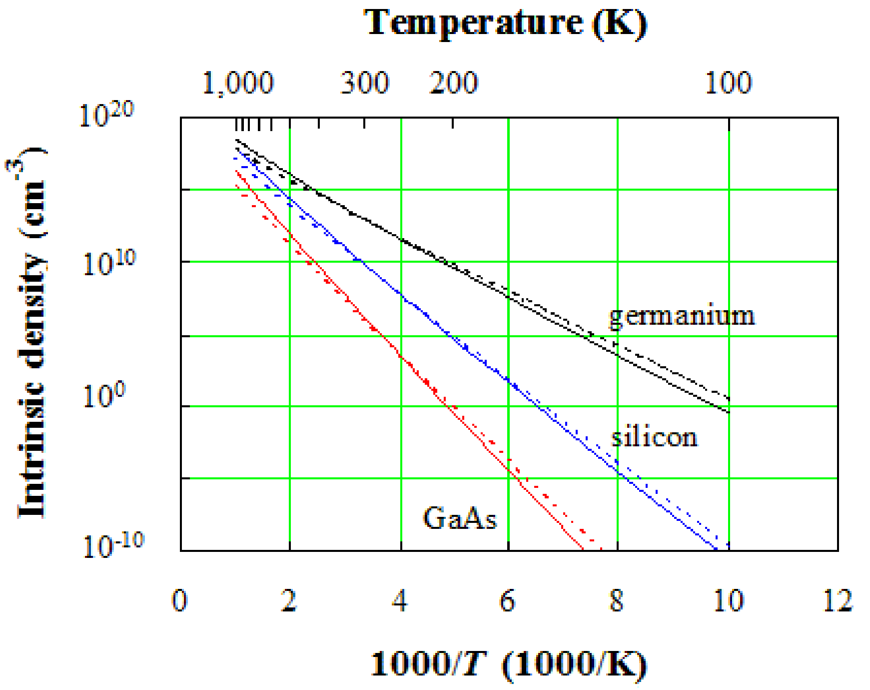
GaAs - direct

Measurement of Band Gaps

temperature dependence
of carrier concentration (or conductivity)

$$n_i \propto T^{3/2} \cdot e^{-E_g/2k_BT}$$

$$\ln n_i \sim -\frac{E_g}{2k_BT}$$



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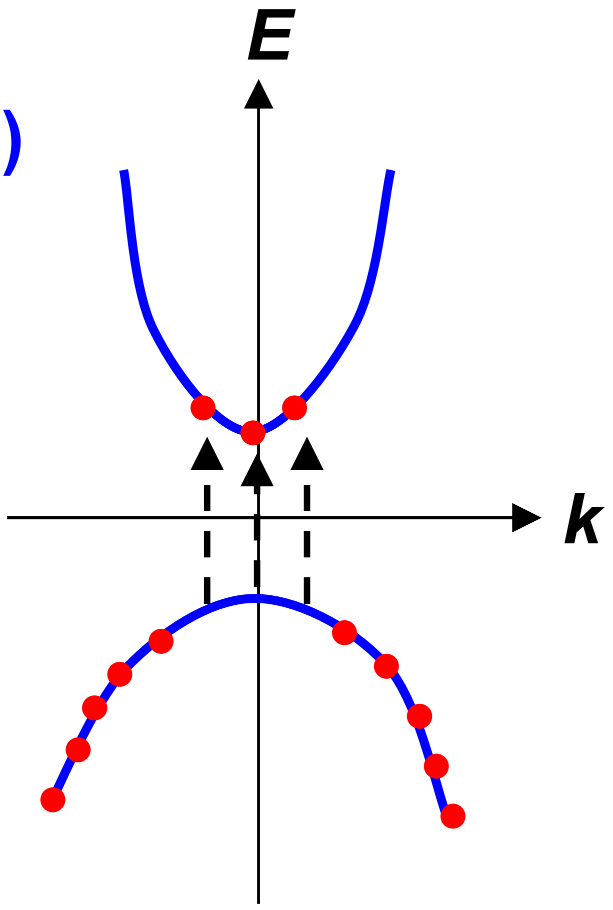
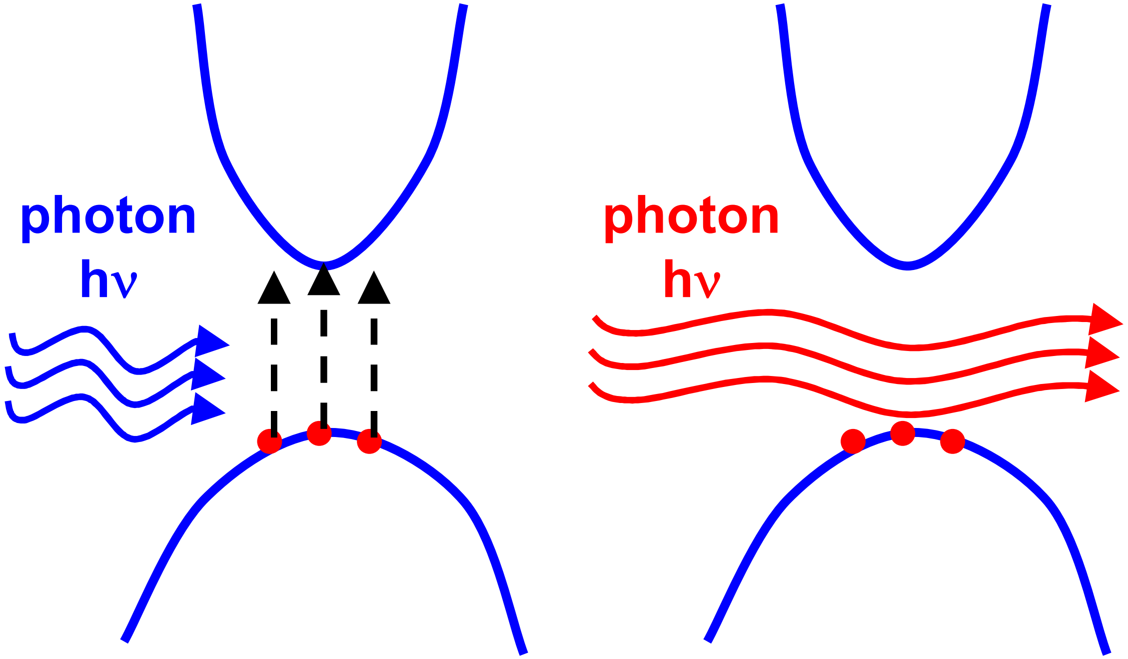


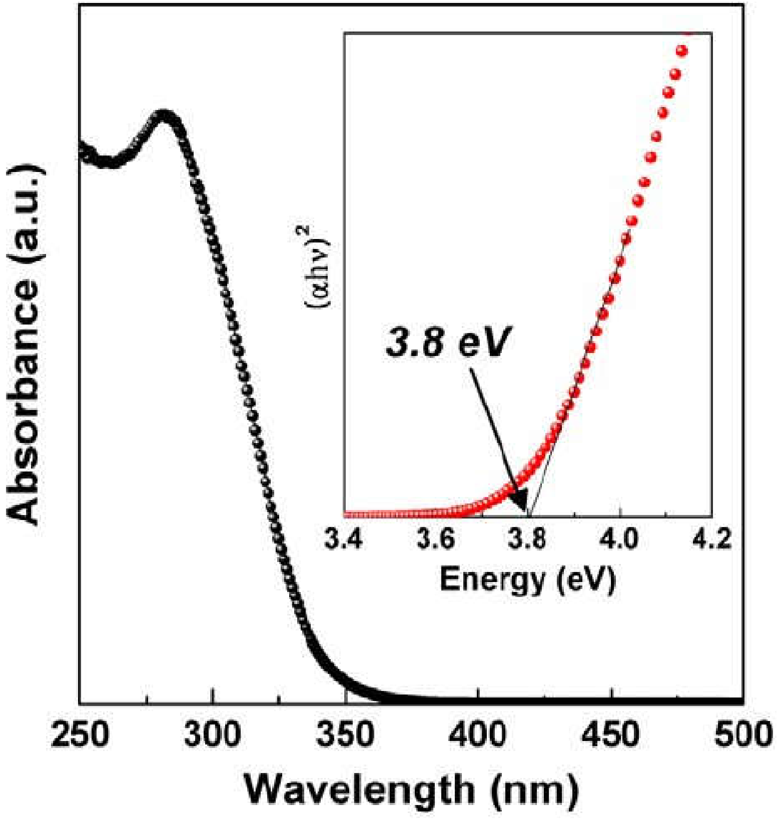
***T* > 0 K**
thermalization 热激发
CB and VB are partly filled
conductor

Measurement of Band Gaps

wavelength dependent optical absorption



Example:
Zn_2SnO_4 nanoparticles

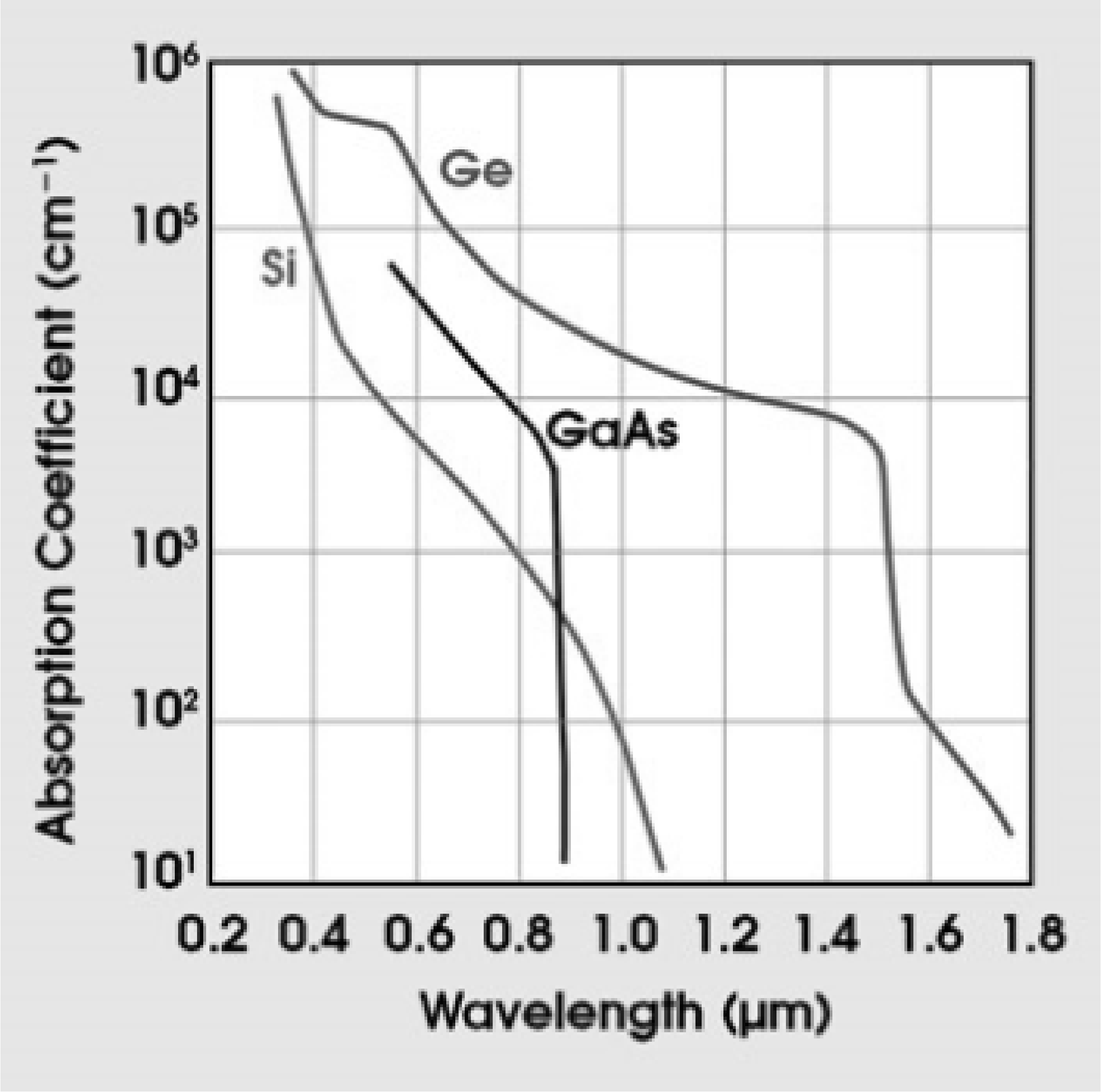


D. Kim, *et al*, *Nanoscale* **4,** 557 (2011)

$h\nu > E_g$ ---> light absorption

$h\nu < E_g$ ---> light transmission

Measurement of Band Gaps



$$\boxed{E_g = \frac{hc}{\lambda_g}} \rightarrow$$

$$\boxed{E(\text{eV}) = \frac{1240}{\lambda(\text{nm})}}$$

	E_g (eV)
Si	**1.1**
Ge	**0.66**
GaAs	**1.43**

Effective Mass 有效质量

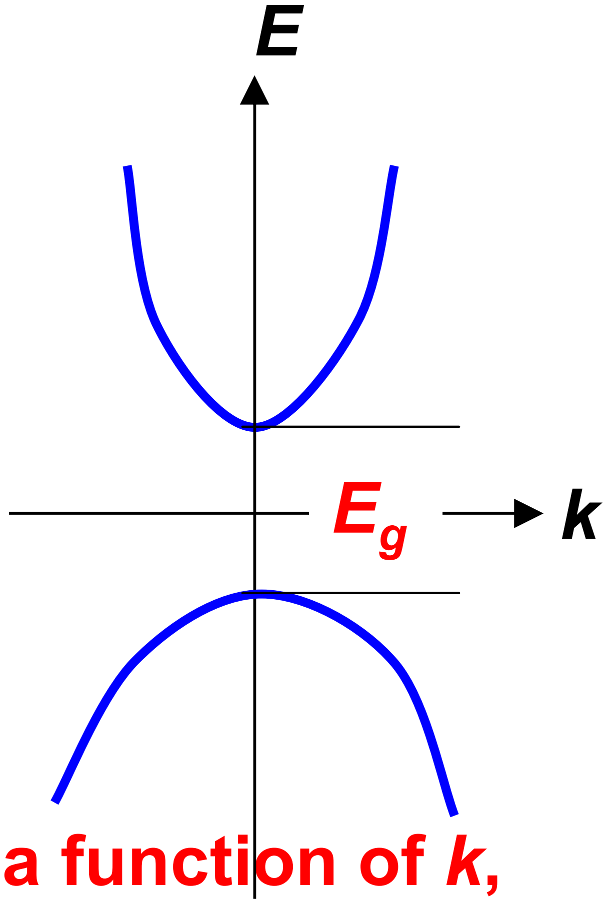
effective mass

$$\frac{1}{m^*}=\frac{1}{\hbar^2}\frac{d^2E(k)}{dk^2}$$

The mass that an electron "seems" to have in a solid.
It has nothing to do with the free electron mass m_0

For 3D solids, a tensor form

$$\frac{1}{\mathbf{M}^*}=\frac{1}{\hbar^2}\begin{pmatrix}\frac{\partial^2E}{\partial k_x^2} & \frac{\partial^2E}{\partial k_x\partial k_y} & \frac{\partial^2E}{\partial k_x\partial k_z}\\ \frac{\partial^2E}{\partial k_y\partial k_x} & \frac{\partial^2E}{\partial k_y^2} & \frac{\partial^2E}{\partial k_y\partial k_z}\\ \frac{\partial^2E}{\partial k_z\partial k_x} & \frac{\partial^2E}{\partial k_z\partial k_y} & \frac{\partial^2E}{\partial k_z^2}\end{pmatrix}$$



m^* is a function of k, can be smaller or larger than m_0, even can be negative

$m_0 = 9.11*10^{-31}$ kg

Effective Mass 有效质量

effective mass

$$\frac{1}{m^*} = \frac{1}{\hbar^2}\frac{d^2E(k)}{dk^2}$$

The mass that an electron "seems" to have in a solid.
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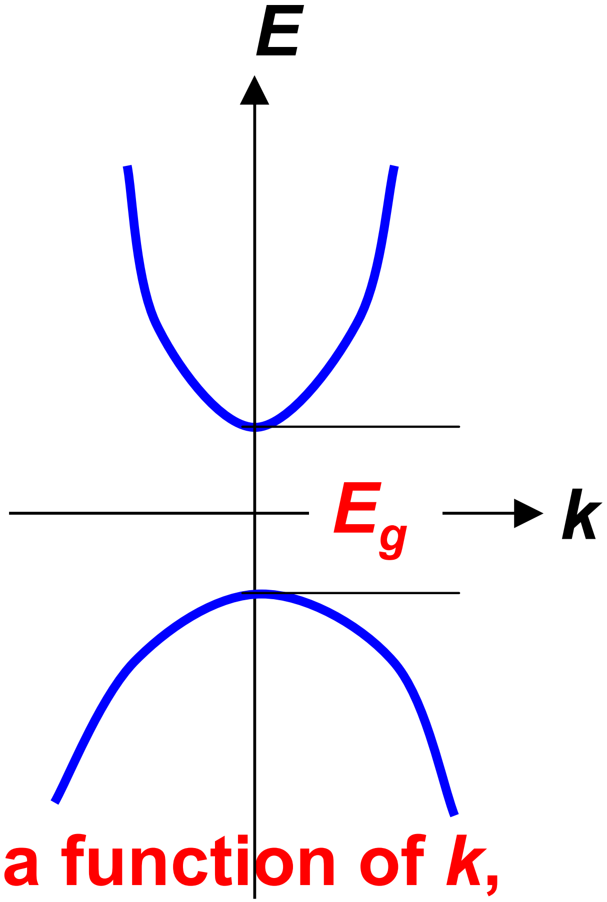
For 3D solids, a tensor form

$$\frac{1}{\mathbf{M}^*_{ij}} = \frac{1}{\hbar^2}\frac{\partial^2 E}{\partial k_i \partial k_j}$$



m^* is a function of k, can be smaller or larger than m_0, even can be negative

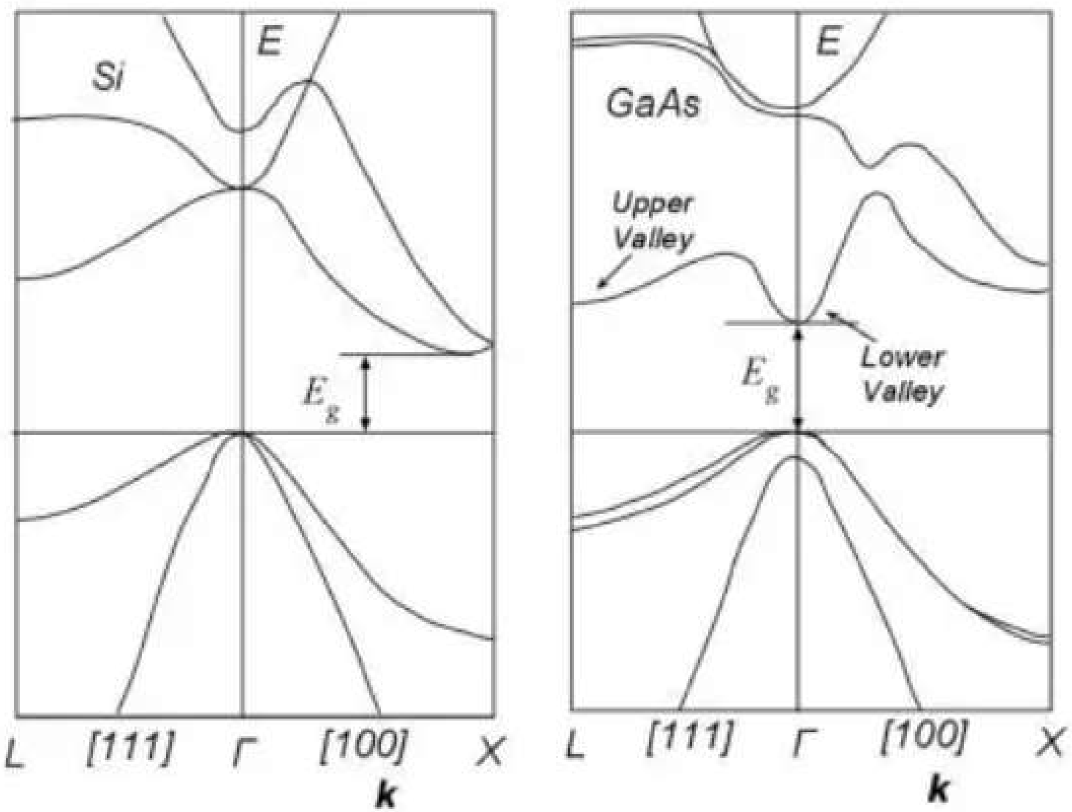
$m_0 = 9.11*10^{-31}$ kg

Effective Mass 有效质量

The actual effective mass is a tensor, depending on the location (k_x, k_y, k_z)

$$\frac{1}{\mathbf{M}_{ij}^*} = \frac{1}{\hbar^2}\frac{\partial^2 E}{\partial k_i \partial k_j}$$

Approximation is taken for different calculations.



Effective Mass 有效质量

The actual effective mass is a tensor, depending on the location (k_x, k_y, k_z)

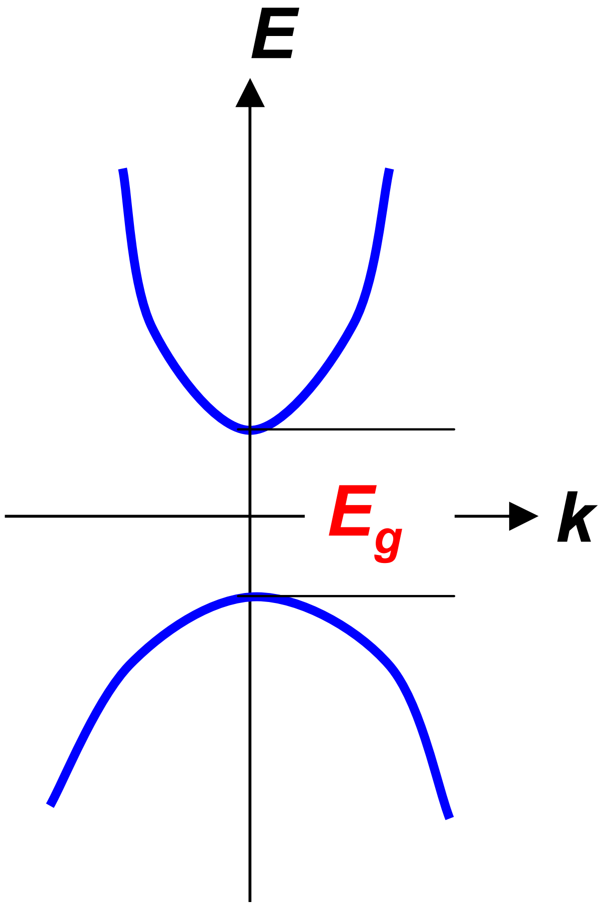
$$\frac{1}{\mathbf{M}^*_{ij}} = \frac{1}{\hbar^2}\frac{\partial^2 E}{\partial k_i \partial k_j}$$

mobility

$$\mu = \frac{v}{E} = e\frac{\tau}{m^*}$$

conductivity

$$\sigma = ne\mu = ne^2\frac{\tau}{m^*}$$



Effective Mass 有效质量

The actual effective mass is a tensor, depending on the location (k_x, k_y, k_z)

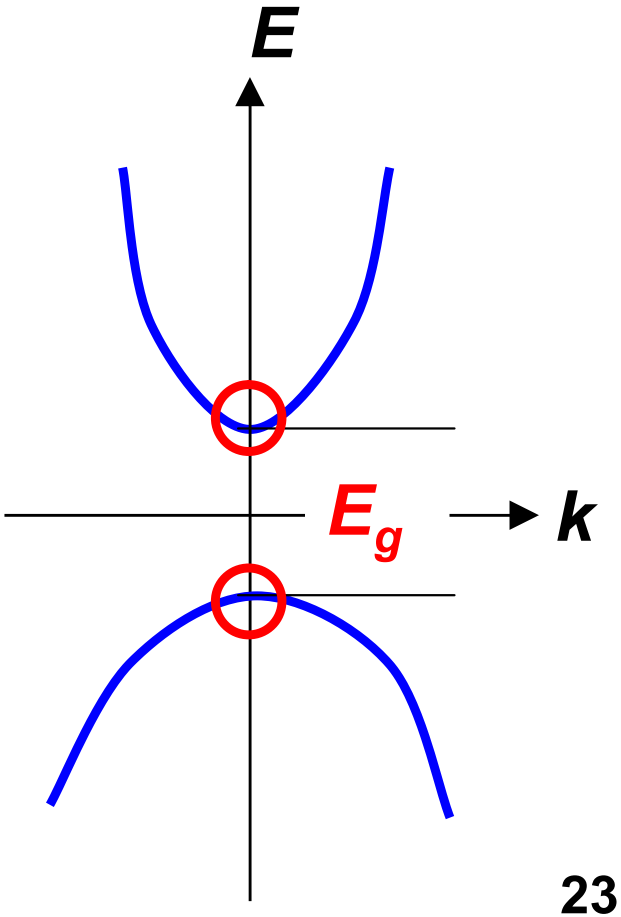
$$\frac{1}{\mathbf{M}^*_{ij}} = \frac{1}{\hbar^2}\frac{\partial^2 E}{\partial k_i \partial k_j}$$

close to band minimum
parabolic approximation

$$E(k) \approx E_0 + \frac{\hbar^2}{2m^*}(k-k_0)^2$$

3D DOS

$$g(E) = \frac{dn}{dE} = \frac{1}{2\pi^2}\left(\frac{2m^*}{\hbar^2}\right)^{3/2}(E-E_0)^{1/2}$$



Lecture Note 3.1

Effective Mass 有效质量

The actual effective mass is a tensor, depending on the location (k_x, k_y, k_z)

$$\frac{1}{\mathbf{M}^*_{ij}} = \frac{1}{\hbar^2}\frac{\partial^2 E}{\partial k_i \partial k_j}$$

Approximation is taken for different calculations:

- Density of states calculations

$$g(E) = \frac{dn}{dE} = \frac{1}{2\pi^2}\left(\frac{2m^*}{\hbar^2}\right)^{3/2}(E - E_0)^{1/2}$$

- Conductivity / mobility calculations

$$\sigma = ne\mu = ne^2\frac{\tau}{m^*}$$

https://ecee.colorado.edu/~bart/book/effmass.htm

Effective Mass 有效质量

The actual effective mass is a tensor, depending on the location (k_x, k_y, k_z)

$$\frac{1}{\mathbf{M}_{ij}^{*}} = \frac{1}{\hbar^2}\frac{\partial^2 E}{\partial k_i \partial k_j}$$

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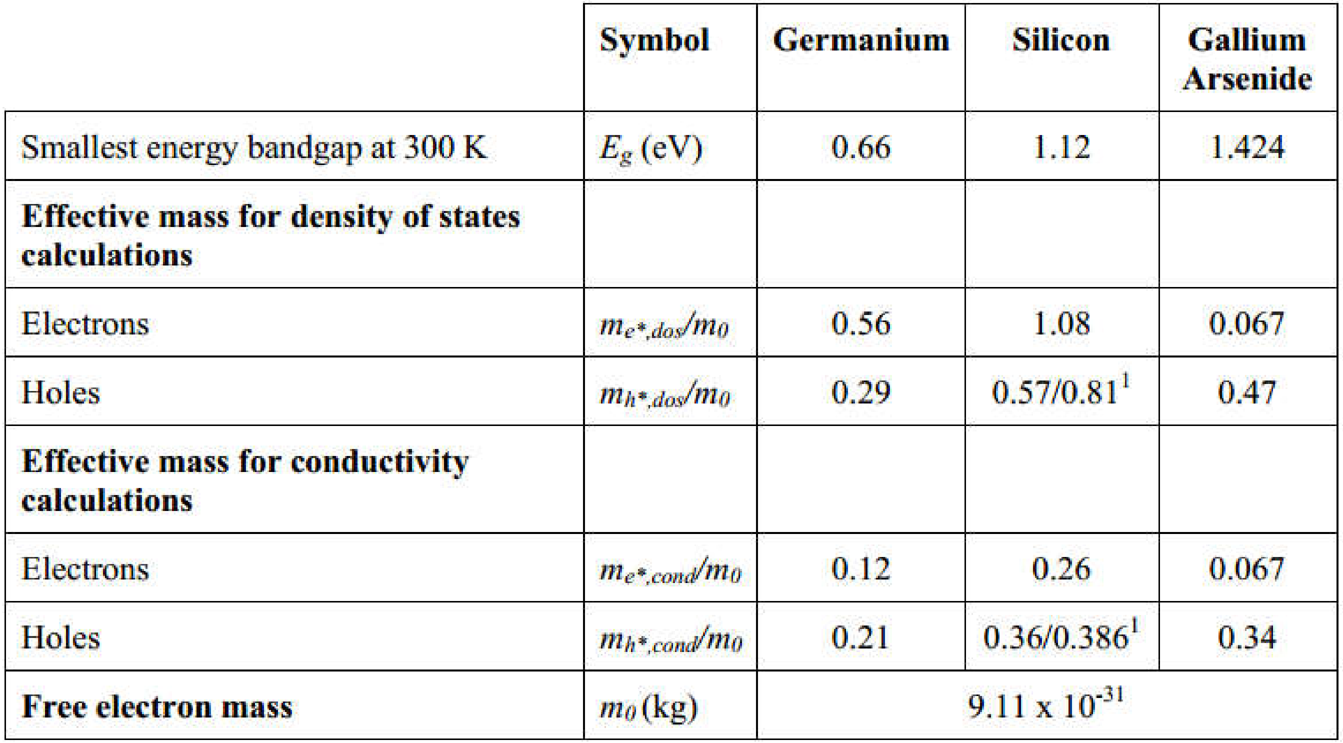


	Symbol	Germanium	Silicon	Gallium Arsenide
Smallest energy bandgap at 300 K	E_g (eV)	0.66	1.12	1.424
Effective mass for density of states calculations				
Electrons	$m_{e^*,dos}/m_0$	0.56	1.08	0.067
Holes	$m_{h^*,dos}/m_0$	0.29	0.57/0.81[1]	0.47
Effective mass for conductivity calculations				
Electrons	$m_{e^*,cond}/m_0$	0.12	0.26	0.067
Holes	$m_{h^*,cond}/m_0$	0.21	0.36/0.386[1]	0.34
Free electron mass	m_0 (kg)	9.11×10^{-31}		

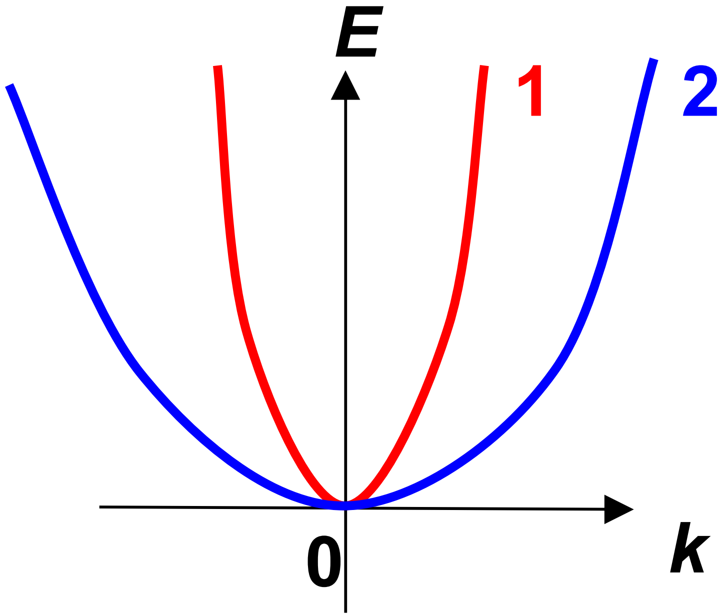
Table 2.3.4 Effective masses for both density of states and conductivity calculations.

Effective Mass 有效质量

effective mass

$$\frac{1}{m^*} = \frac{1}{\hbar^2}\frac{d^2E(k)}{dk^2}$$

inverse curvature of the parabolic curve

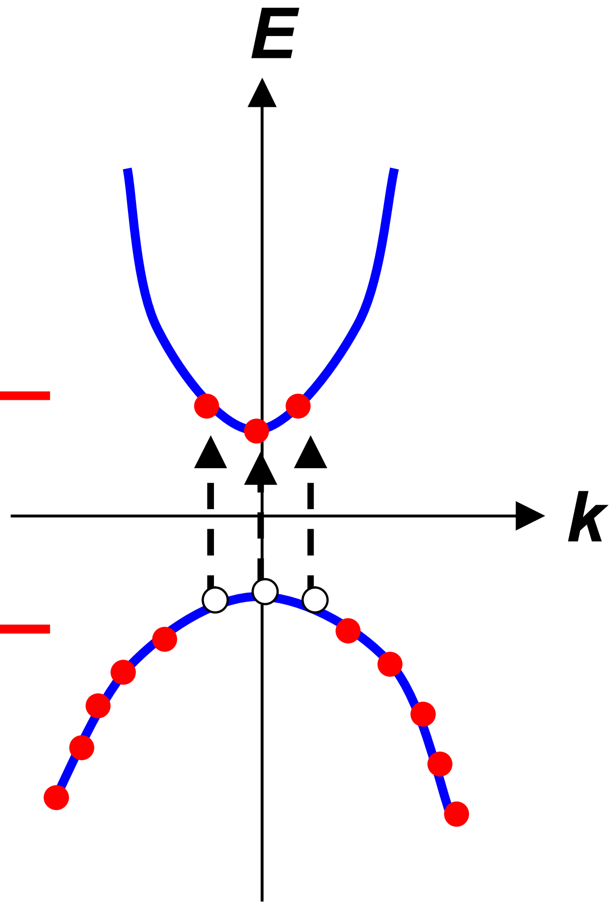


Q: $m_1 > m_2$
or $m_1 < m_2$?

Effective Mass 有效质量

effective mass

$$\boxed{\frac{1}{m^*} = \frac{1}{\hbar^2}\frac{d^2E(k)}{dk^2}}$$

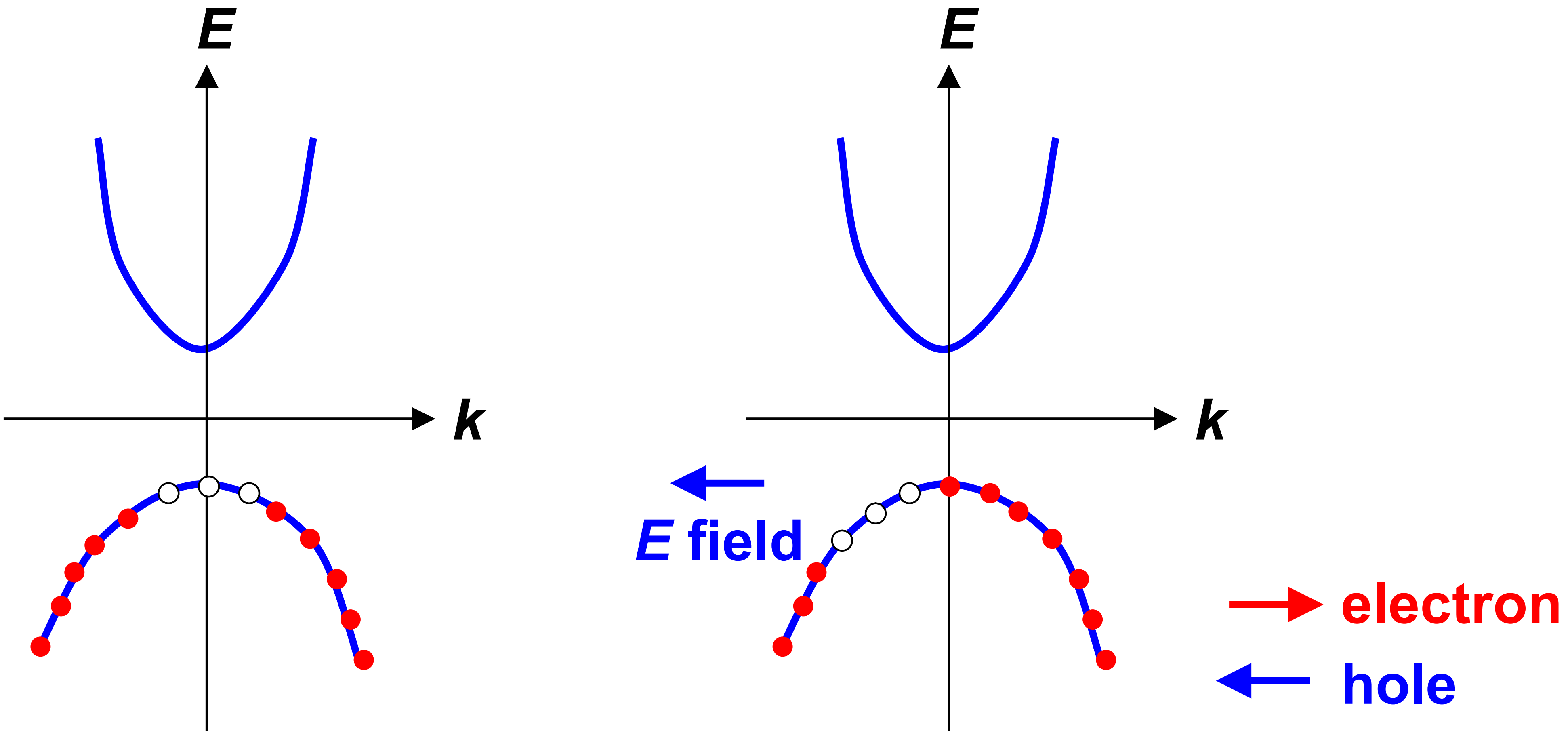


$\boxed{m_e^* > 0}$ ←

$\boxed{m_e^* < 0}$ ←

Hole 空穴

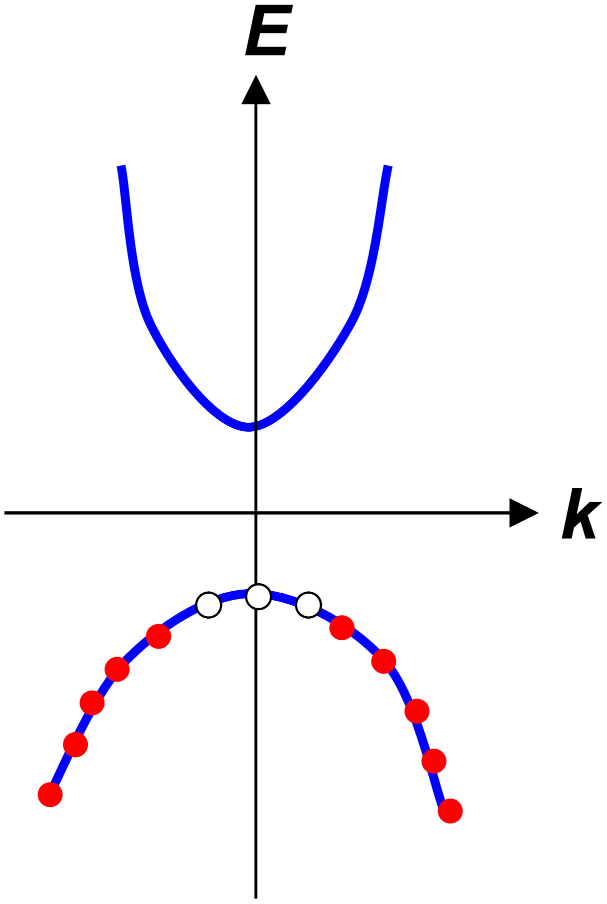
We conventionally use *holes* to analyze the electron behaviors in VB

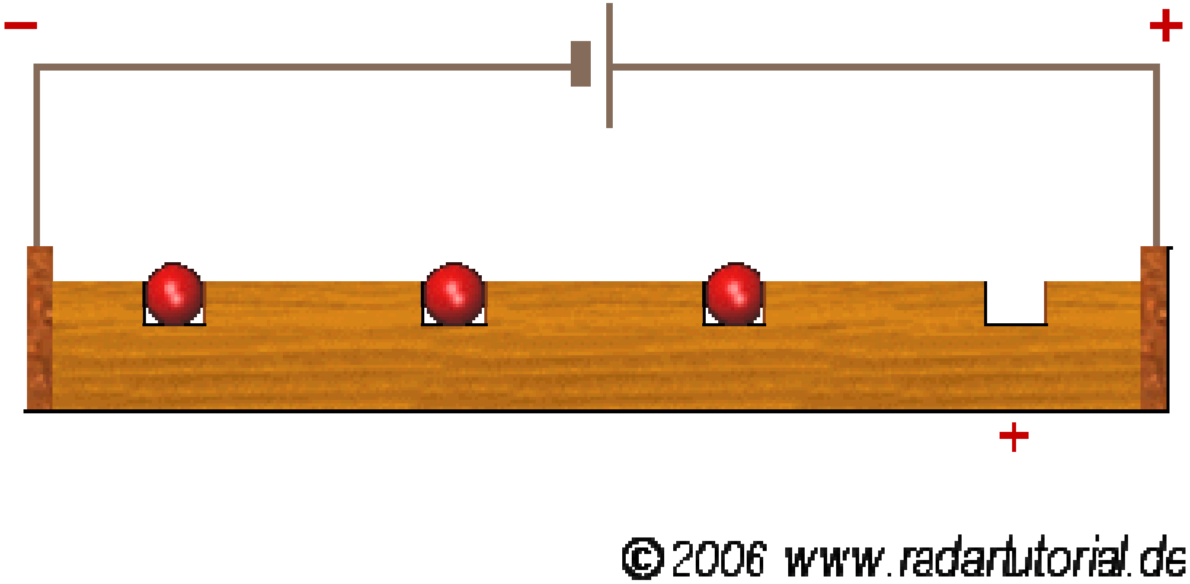


hole is the absence of an electron

Hole 空穴

We conventionally use *holes* to analyze the electron behaviors in VB

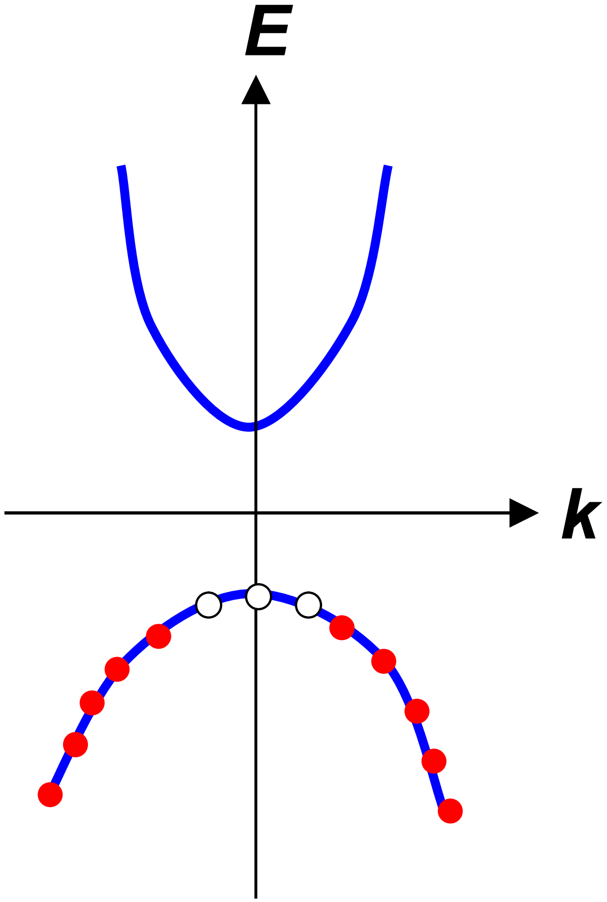


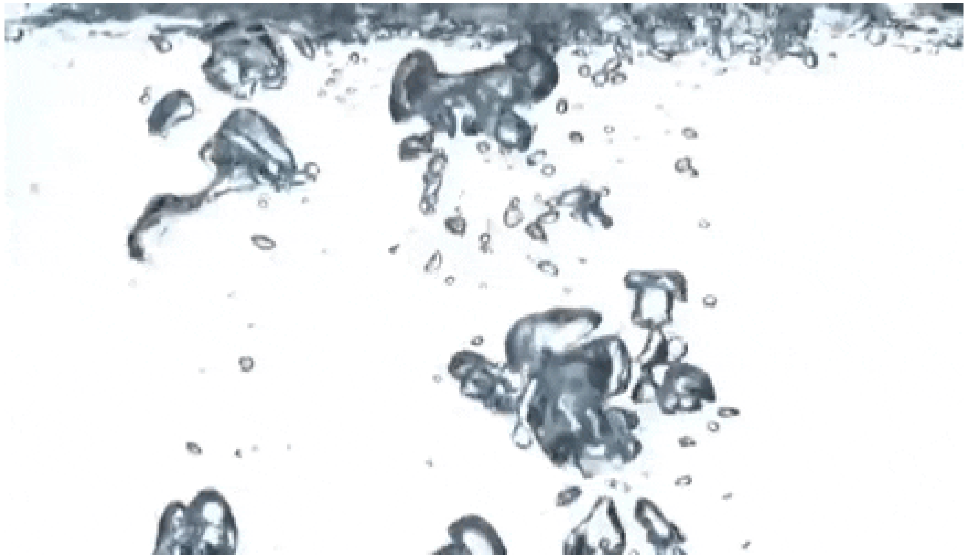


hole is a quasi-particle (准粒子), different from positron (正电子)

Hole 空穴

We conventionally use *holes* to analyze the electron behaviors in VB

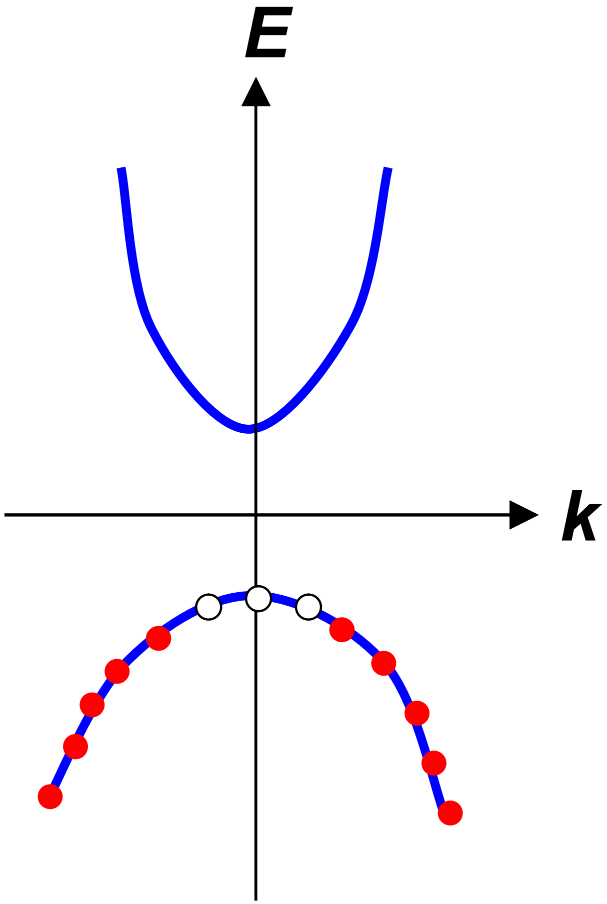




air bubbles in water

Hole 空穴

In VB, properties of a hole compared to a missing electron in the same position of the band



charge $q_h = -e$

wavevector $\mathbf{k}_h = -\mathbf{k}_e$

energy $E_h = -E_e$

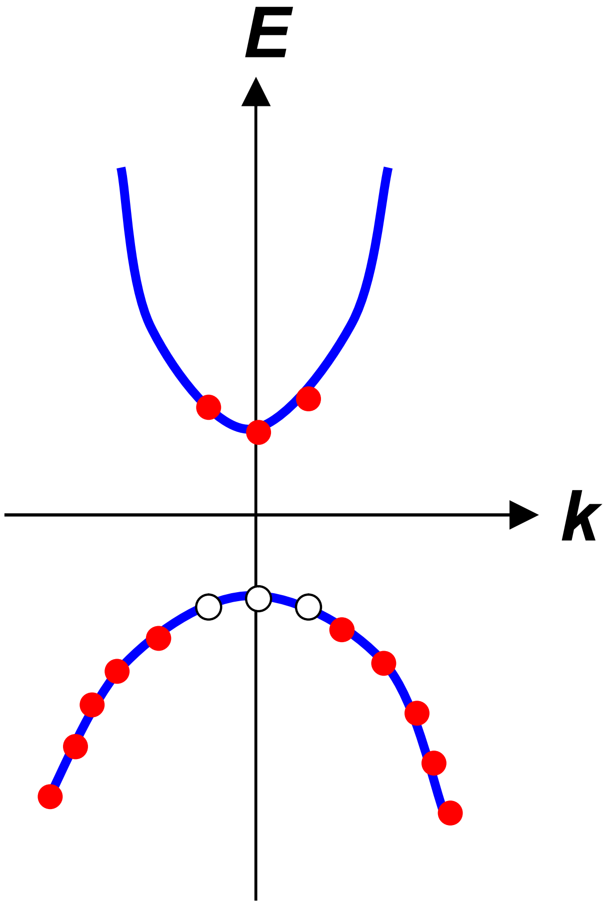
group velocity $\mathbf{v}_h = \mathbf{v}_e$

effective mass $m_h^* = -m_e^*$

Singleton, p45-p46

Carriers 载流子

Particles that conduct electrical current: electrons in CB and holes in VB



in CB

$$m_e^* > 0$$

electron mobility

$$\mu_e = \frac{e^2 \tau}{m_e^*}$$

in VB

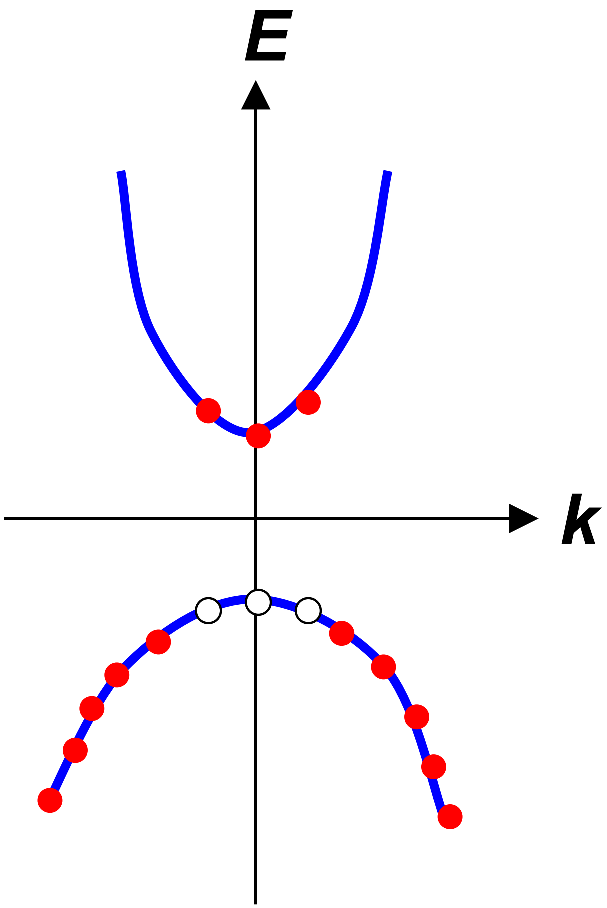
$$m_h^* > 0$$

hole mobility

$$\mu_h = \frac{e^2 \tau}{m_h^*}$$

Carriers 载流子

Particles that conduct electrical current: electrons in CB and holes in VB



Q: How to calculate:

m_e^* in CB

m_h^* in VB?

The Nearly Free Electron Model

$$\begin{vmatrix} \frac{\hbar^2}{2m}(k-g)^2 - E & -V_1 \\ -V_1 & \frac{\hbar^2}{2m}k^2 - E \end{vmatrix} = 0$$

$$\rightarrow \boxed{\left[\frac{\hbar^2}{2m}(k-g)^2 - E\right]\left[\frac{\hbar^2}{2m}k^2 - E\right] - V_1^2 = 0}$$

$$\rightarrow \boxed{E_1(k), E_2(k)}$$

The Nearly Free Electron Model

Nearly Free electron → $\boxed{V_1 \neq 0}$ **and** $\boxed{V_1 \ll \frac{\hbar^2}{2m}\left(\frac{\pi}{a}\right)^2}$

$$\left[\frac{\hbar^2}{2m}(k-g)^2 - E\right]\left[\frac{\hbar^2}{2m}k^2 - E\right] - V_1^2 = 0$$

→ $$\boxed{E_\pm(k) = \frac{(A+B) \pm \sqrt{(A-B)^2 + 4V_1^2}}{2}}$$

$$\boxed{A = \frac{\hbar^2}{2m}k^2} \qquad \boxed{B = \frac{\hbar^2}{2m}(k-g)^2}$$

The Nearly Free Electron Model

Nearly Free electron → $\boxed{V_1 \neq 0}$ **and** $\boxed{V_1 \ll \frac{\hbar^2}{2m}\left(\frac{\pi}{a}\right)^2}$

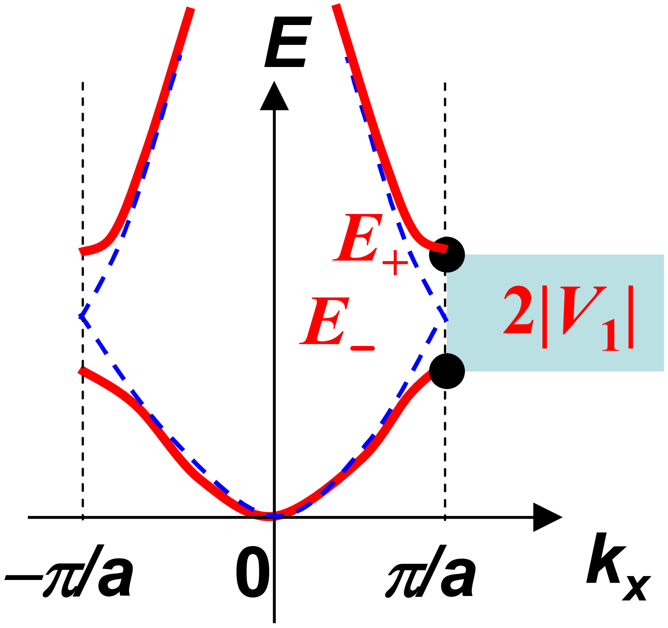
$$\left[\frac{\hbar^2}{2m}(k-g)^2 - E\right]\left[\frac{\hbar^2}{2m}k^2 - E\right] - V_1^2 = 0$$

when $k = \pi/a$

$$E_+\left(k = \frac{\pi}{a}\right) = \frac{\hbar^2}{2m}\left(\frac{\pi}{a}\right)^2 + |V_1|$$

→

$$E_-\left(k = \frac{\pi}{a}\right) = \frac{\hbar^2}{2m}\left(\frac{\pi}{a}\right)^2 - |V_1|$$



Band Gap E_g!

The Nearly Free Electron Model

Nearly Free electron → $\boxed{V_1 \neq 0}$

$$\left[\frac{\hbar^2}{2m}(k-g)^2 - E\right]\left[\frac{\hbar^2}{2m}k^2 - E\right] - V_1^2 = 0$$

when *k* ~ *π/a*, *(A–B)* ~ 0, take the first order approximation

$$\boxed{E_\pm(k) = \frac{(A+B) \pm \sqrt{(A-B)^2 + 4V_1^2}}{2}}$$

→ $$\boxed{E_\pm(k) \approx \frac{A+B}{2} \pm V_1\left[1 + \frac{1}{2}\frac{(A-B)^2}{4V_1^2}\right]}$$

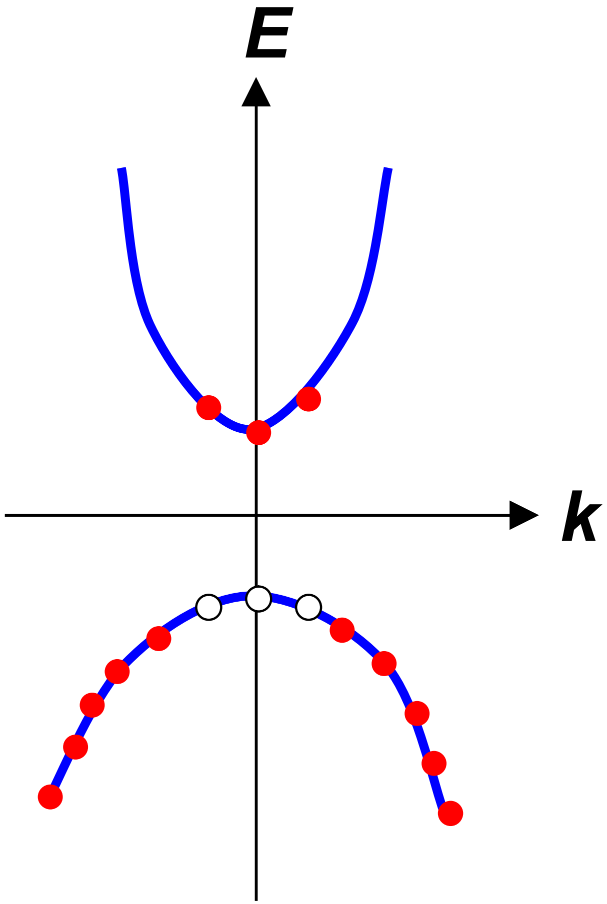
Effective Mass 有效质量

$$\boxed{E_{\pm}(k) \approx \frac{A+B}{2} \pm V_1\left[1+\frac{1}{2}\frac{(A-B)^2}{4V_1^2}\right]}$$

$$\boxed{\frac{1}{m^*} = \frac{1}{\hbar^2}\frac{d^2E(k)}{dk^2}}$$

$$\rightarrow \begin{cases} \dfrac{m_e^*}{m_0} \approx \dfrac{1}{C/V_1+1} \\[2ex] \dfrac{m_h^*}{m_0} \approx \dfrac{1}{C/V_1-1} \end{cases}$$

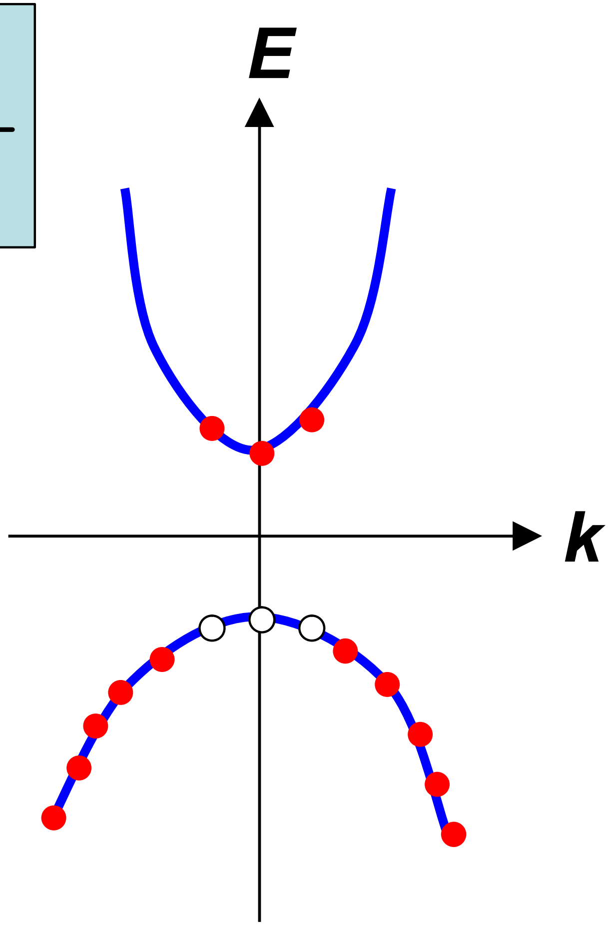
$$\boxed{C = \frac{\hbar^2 g^2}{4m}}$$



Effective Mass 有效质量

$$\begin{cases} \dfrac{m_e^*}{m_0} \approx \dfrac{1}{C/V_1 + 1} \\ \dfrac{m_h^*}{m_0} \approx \dfrac{1}{C/V_1 - 1} \end{cases}$$

$$\boxed{\mu = e\frac{\tau}{m^*}}$$



For many semiconductors, V_1 is very small (C/V_1 ~ 1–10)

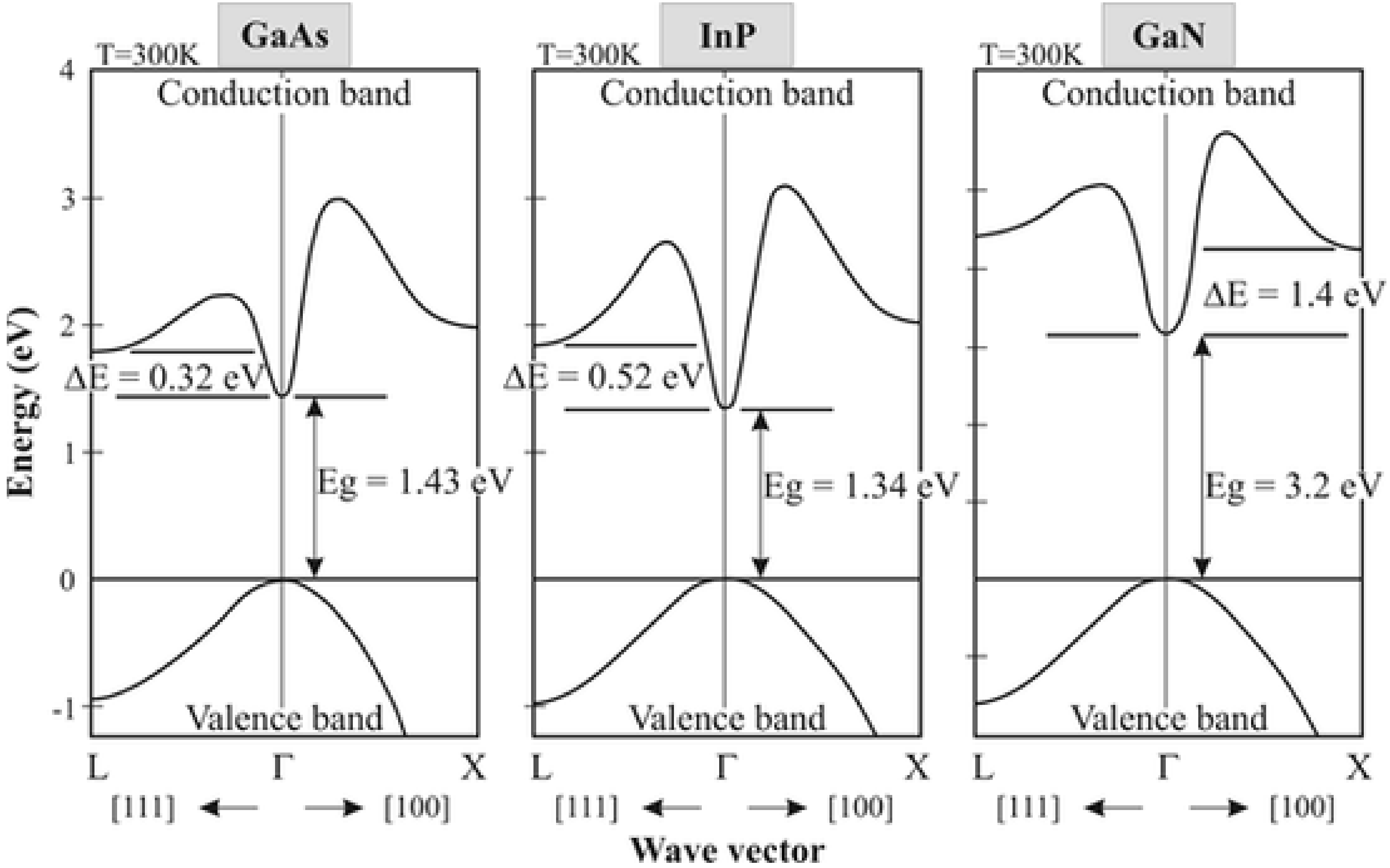
$$\rightarrow \boxed{m_e^* < m_h^* < m_0}$$

- Effective mass is smaller than real electron mass m_0

- m_e* in CB is smaller than m_h* in VB

- small V_1 ----> small m* ----> large mobility μ

Examples



m_e^* in CB is smaller than m_h^* in VB

(electrons have more freedom than holes)

Examples

	a (Å)	E_g (eV)	m_e^* / m_0	m_h^* / m_0	μ_e (cm²/V/s)	μ_h (cm²/V/s)
Si	5.43	1.1	0.26	0.38	1350	450
Ge	5.66	0.66	0.12	0.23	3900	1900
-	-	-	-	-	-	-
GaAs	5.65	1.42	0.067	0.45	8500	400
InAs	6.06	0.35	0.022	0.40	33000	450

* effective mass for conductivity

1. large atoms ----> small V_1 ----> small E_g

----> small m ----> large mobility μ*

2. $m_e^* < m_h^* < m_0$

3. $\mu_e^* > \mu_h^*$

Examples

	a (Å)	E_g (eV)	m_e^* / m_0	m_h^* / m_0	μ_e (cm²/V/s)	μ_h (cm²/V/s)
Si	5.43	1.1	0.26	0.38	1350	450
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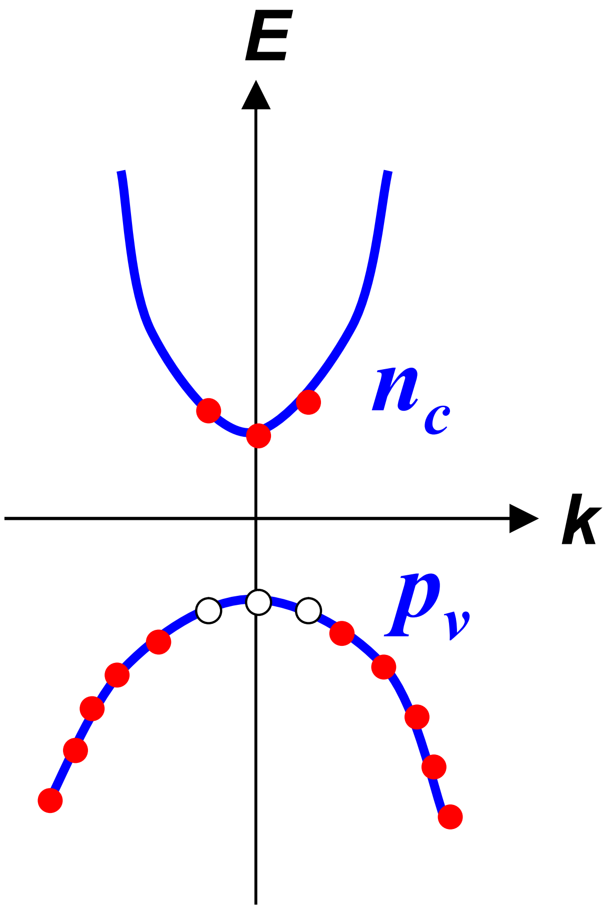
-	-	-	-	-	-	-
GaAs	5.65	1.42	0.067	0.45	8500	400
InAs	6.06	0.35	0.022	0.40	33000	450

* effective mass for conductivity

larger atoms
----> electrons have more freedom
----> smaller m, move faster*

Carriers 载流子

Particles that conduct electrical current: electrons in CB and holes in VB



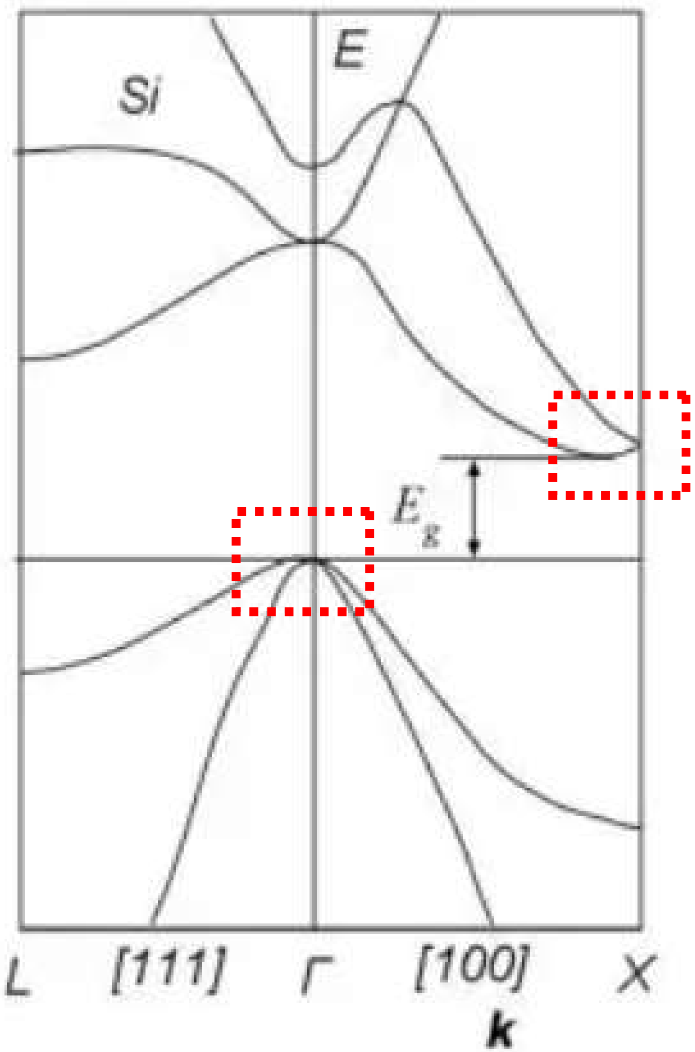
electrical conductivity

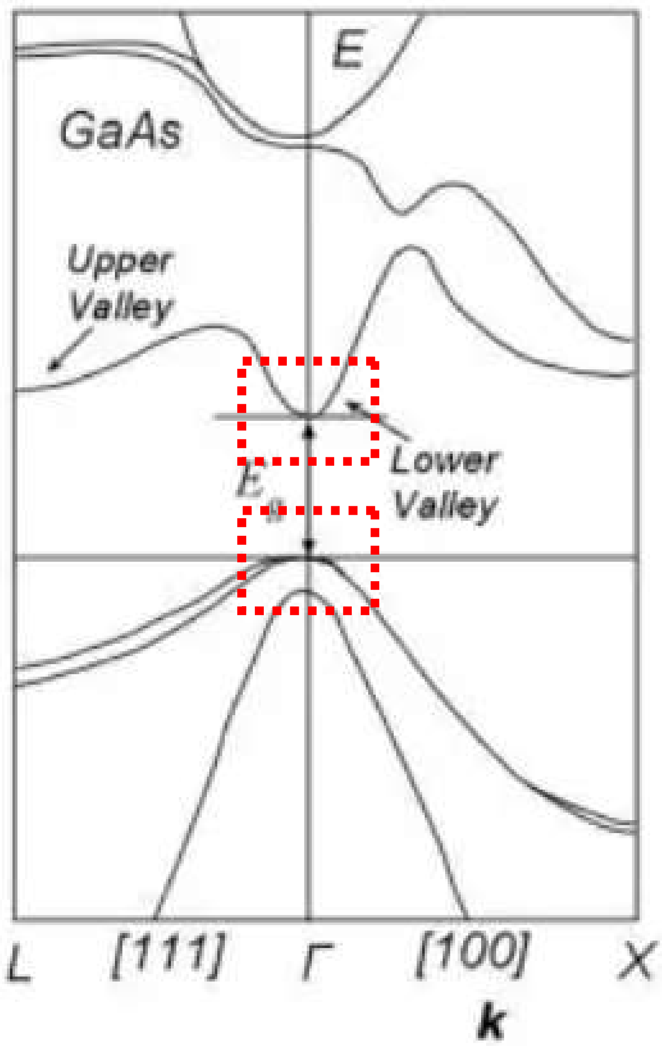
$$\boxed{\sigma = n_c e \mu_e + p_v e \mu_h}$$

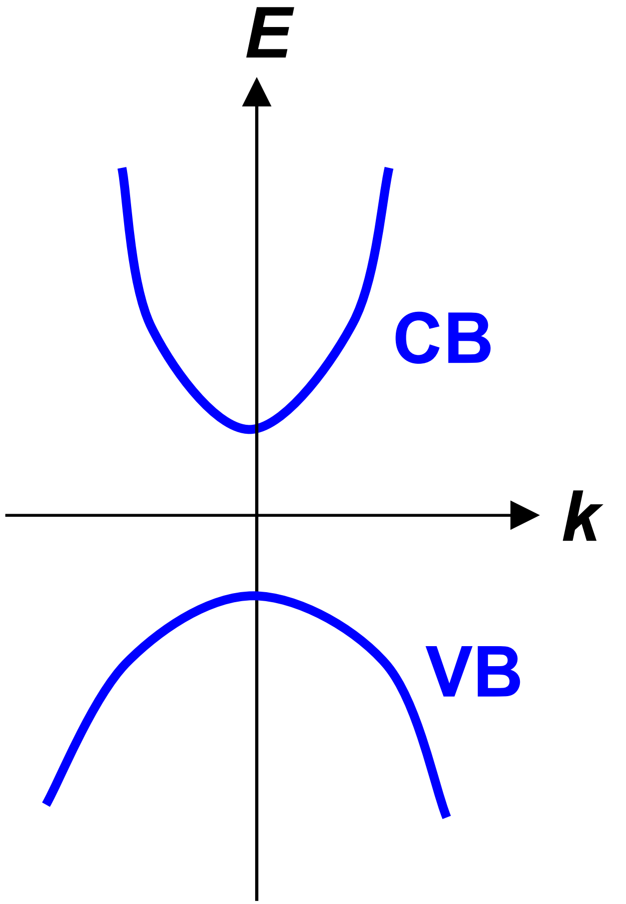
Q: How to calculate carrier densities?

n_c and p_v (#/cm^3)

Band Diagram of Semiconductors







The peaks and valleys of VB and CB can be approximately by *parabolic functions*

Band Diagram of Semiconductors

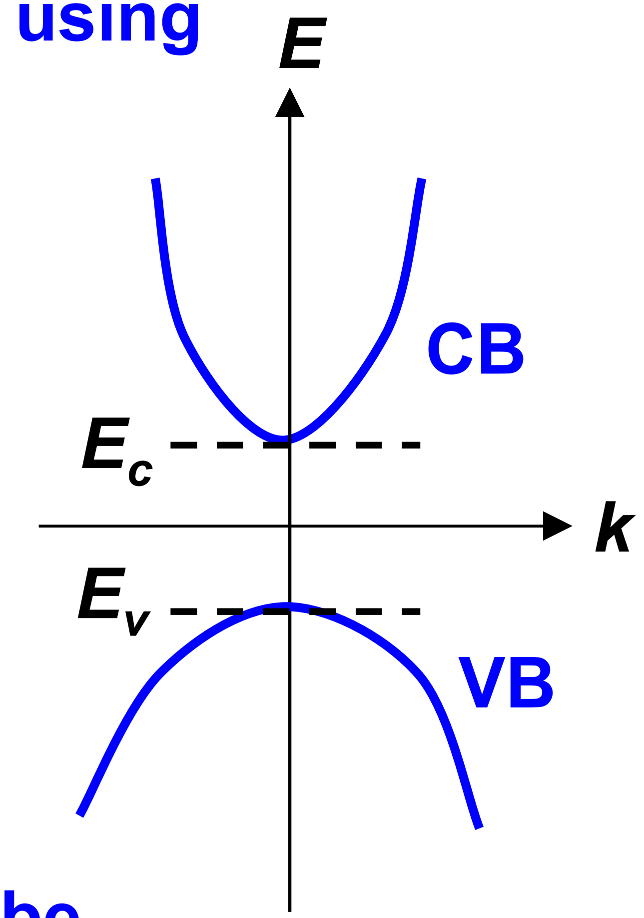
electrons and holes can be approximated using *free electron gas*

- modify the Sommerfeld Model

$$E(k) = E_c + \frac{\hbar^2 k^2}{2m_e^*}$$

$$E_c - E_v = E_g$$

$$E(k) = E_v - \frac{\hbar^2 k^2}{2m_h^*}$$



The peaks and valleys of VB and CB can be approximately by *parabolic functions*

Q: How many electrons and holes?

Density of States (DOS) 态密度

$$g(E) = \frac{dn}{dE}$$

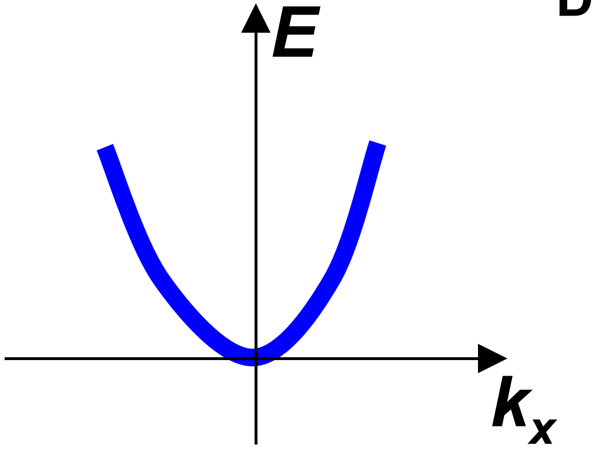
DOS - number of energy states/levels per unit energy in [*E*, *E*+*dE*], per unit volume

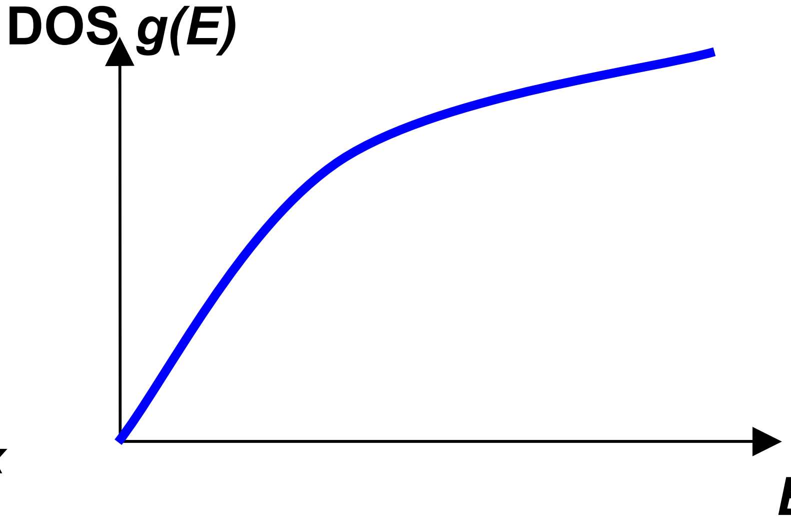
free electrons in 3D

$$E = \frac{\hbar^2 k^2}{2m_e}$$

$$g(E) = \frac{dn}{dE} = \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2} E^{1/2}$$

*replace mass m with effective mass m**





Lecture Note 3.1

Density of States (DOS) 态密度

$$g(E) = \frac{dn}{dE}$$

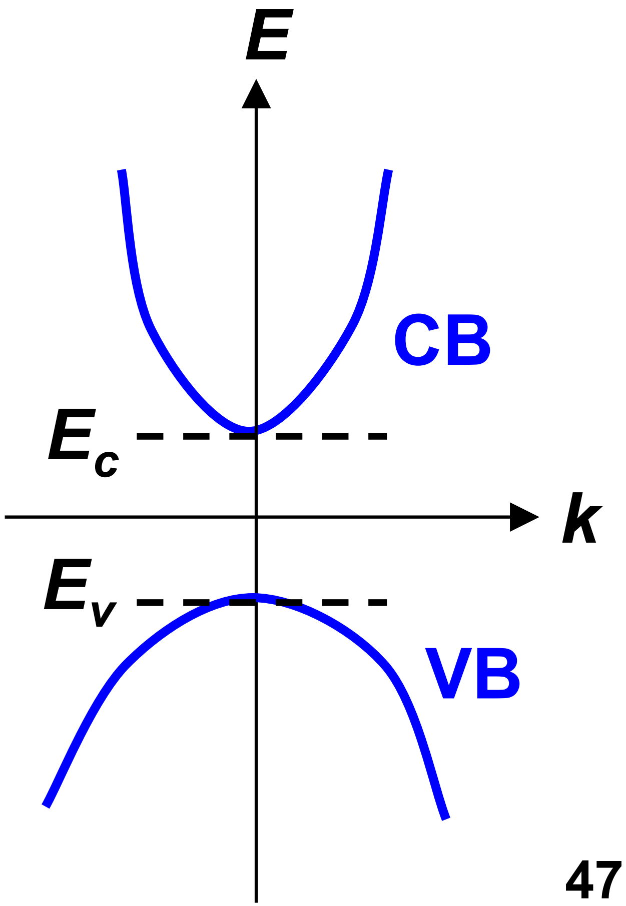
DOS - number of energy states/levels per unit energy in [*E*, *E*+*dE*], per unit volume

DOS for electrons in CB

$$g_c(E) = \frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{3/2}(E - E_c)^{1/2}$$

DOS for holes in VB

$$g_v(E) = \frac{1}{2\pi^2}\left(\frac{2m_h^*}{\hbar^2}\right)^{3/2}(E_v - E)^{1/2}$$



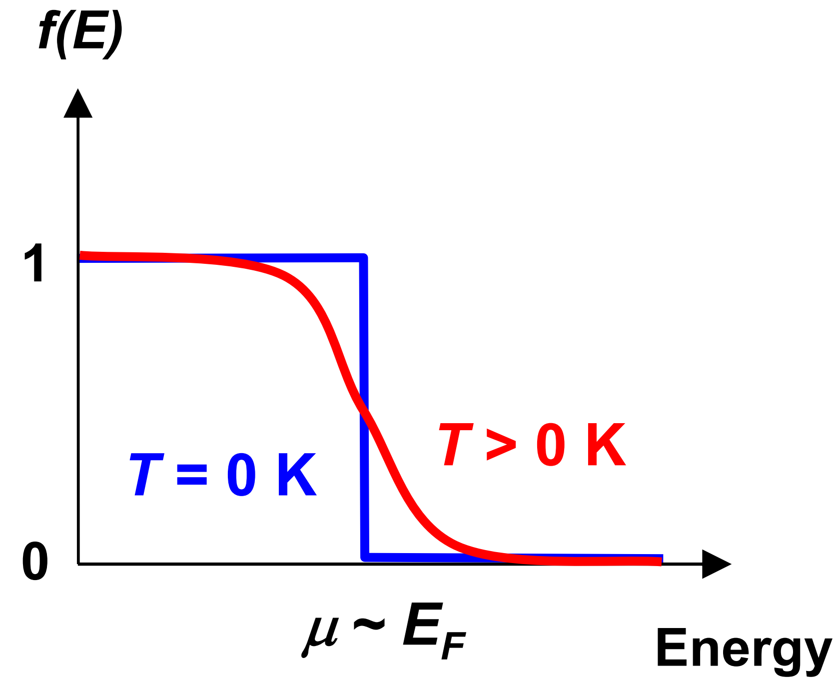
Density of Carriers

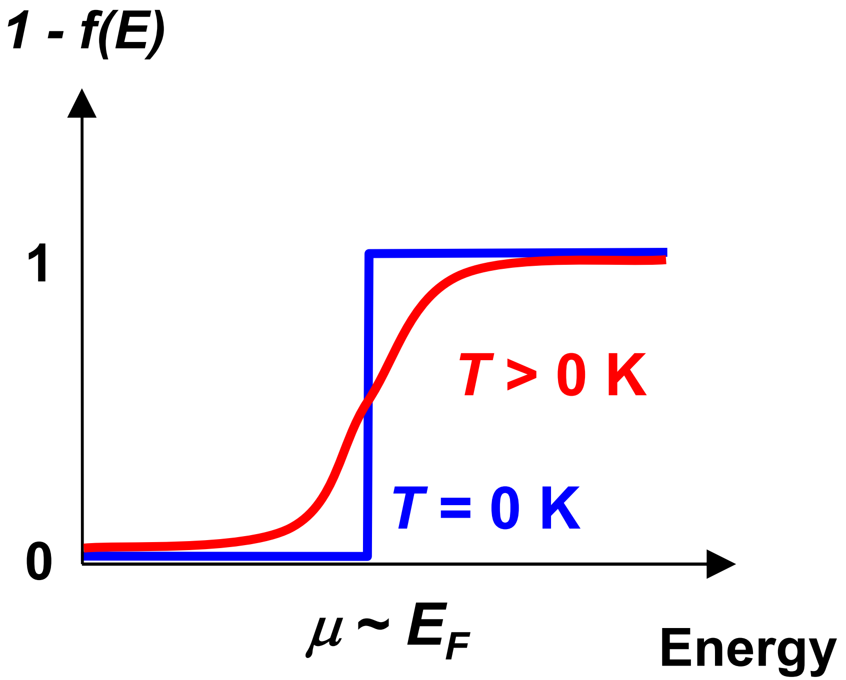
Density of electrons = DOS * probability f

Density of holes = DOS * $(1-f)$

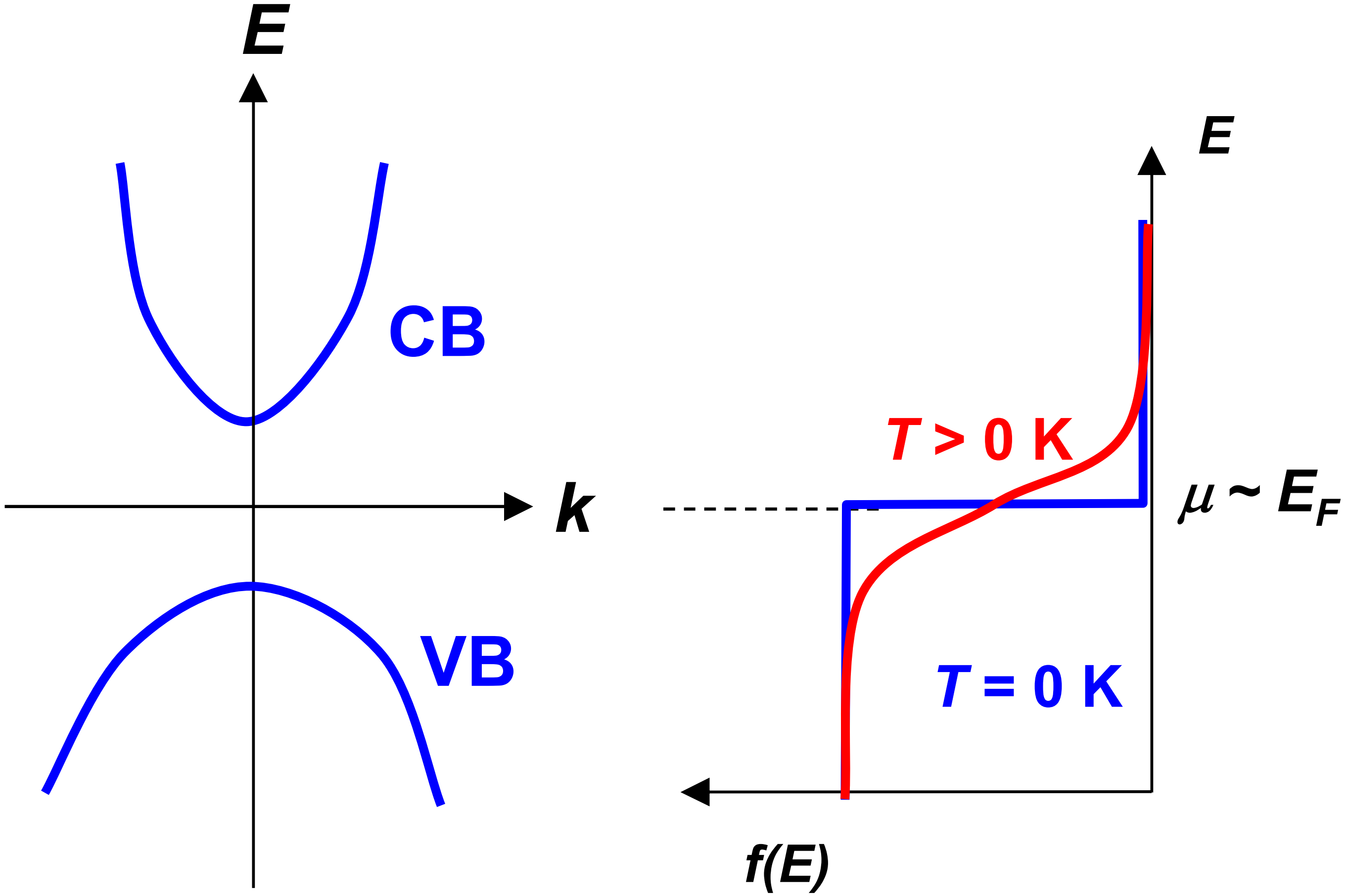
$$f(E)=\frac{1}{e^{(E-\mu)/k_BT}+1}$$

$$1-f(E)=1-\frac{1}{e^{(E-\mu)/k_BT}+1}=\frac{1}{e^{(\mu-E)/k_BT}+1}$$





Chemical Potential



For pure semiconductors (intrinsic), the chemical potential μ (Fermi level E_F) lie within the band gap.

Fermi Energy E_F - A Little Note

In metals, Fermi energy/level E_F is the highest occupied state of electrons at $T = 0$ K.

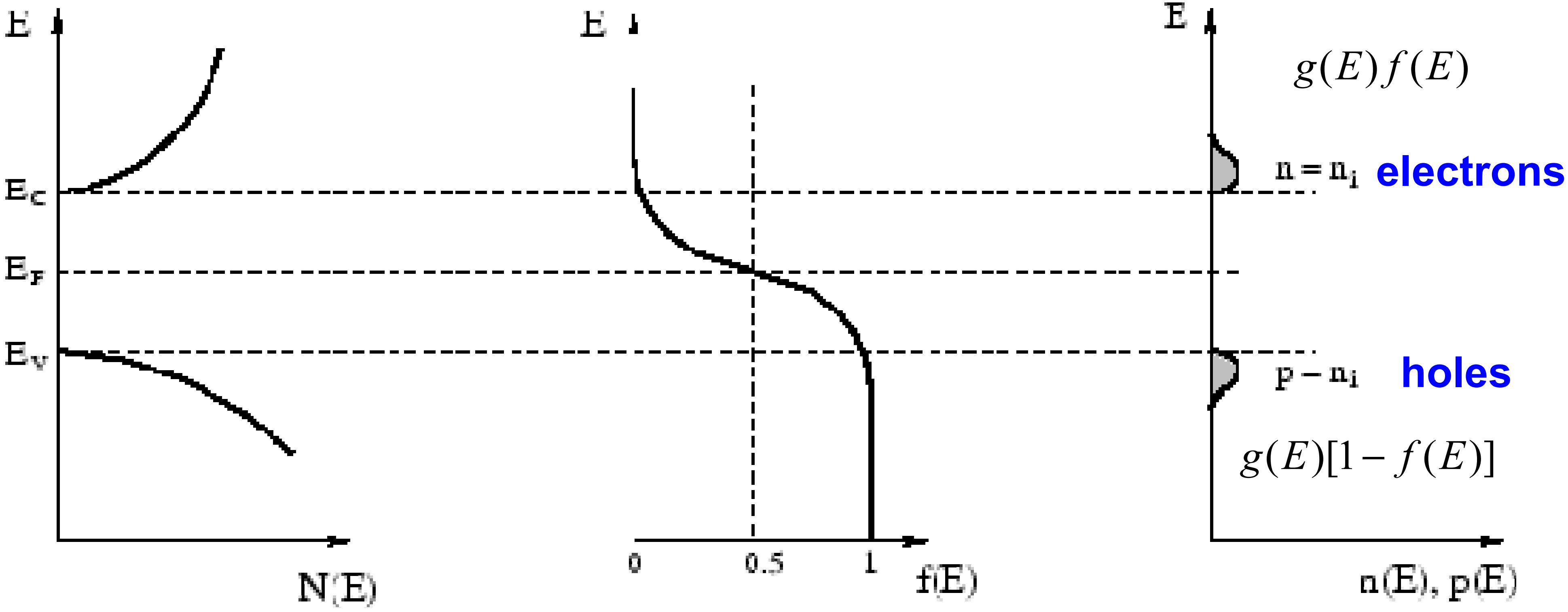
In semiconductors, Fermi energy/level E_F is referred to the chemical potential μ, which is inside the gap. No electrons at E_F !

> ***"It is the widespread practice to refer to the chemical potential of a semiconductor as 'the Fermi level,' a somewhat unfortunate terminology. ... The term 'Fermi level' should be regarded as nothing more than a synonym for 'chemical potential,' in the context of semiconductors."***
>
> ***---- Ashcroft & Mermin, p573***

Density of Carriers = DOS * Probability

Intrinsic

$g(E)$ $f(E)$



electrons

holes

DOS

electron probability

carriers

Density of Carriers

electrons in CB

$$n_c = \int_{E_c}^{+\infty} g_c(E) \cdot f(E) dE$$

If μ is in the gap, assume

$$E_c - \mu \gg k_B T$$

→

$$f(E) = \frac{1}{e^{(E-\mu)/k_B T} + 1} \approx e^{-(E-\mu)/k_B T}$$

holes in VB

$$p_v = \int_{-\infty}^{E_v} g_v(E) \cdot [1 - f(E)] dE$$

$$\mu - E_v \gg k_B T$$

→

$$1 - f(E) = \frac{1}{e^{(\mu-E)/k_B T} + 1} \approx e^{-(\mu-E)/k_B T}$$

Non-Degenerate semiconductors (非简并半导体):
Fermi-Dirac is approximated by Maxwell-Boltzmann distribution
not valid for high temperature or small band gap

Density of Carriers

electrons in CB

$$n_c = \int_{E_c}^{+\infty} g_c(E)\cdot f(E)dE$$

$$= \int_{E_c}^{+\infty} \frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{3/2} (E-E_c)^{1/2}\cdot e^{-(E-\mu)/k_BT}dE$$

$$= \frac{1}{4}\left(\frac{2m_e^* k_B T}{\pi\hbar^2}\right)^{3/2} e^{-(E_c-\mu)/k_BT}$$

$$= N_c(T)e^{-(E_c-\mu)/k_BT}$$

$$N_c(T) = \frac{1}{4}\left(\frac{2m_e^* k_B T}{\pi\hbar^2}\right)^{3/2} = 2.5\left(\frac{m_e^*}{m_0}\right)^{3/2}\left(\frac{T}{300\ \mathrm{K}}\right)^{3/2}\times 10^{19}\,\mathrm{cm}^{-3}$$

Density of Carriers

note here we use the integral

$$\int_0^{+\infty} x^{1/2} \cdot e^{-x/a} dx = \frac{\sqrt{\pi}}{2} a^{3/2}$$

so

$$\int_{E_c}^{+\infty} (E - E_c)^{1/2} \cdot e^{-(E-\mu)/k_B T} dE$$

$$= \frac{\sqrt{\pi}}{2} (k_B T)^{3/2} e^{-(E_c - \mu)/k_B T}$$

Density of Carriers

holes in VB

$$
\begin{aligned}
p_v &= \int_{-\infty}^{E_v} g_v(E)\cdot\left[1-f(E)\right]dE \\
&= \int_{-\infty}^{E_v} \frac{1}{2\pi^2}\left(\frac{2m_h^*}{\hbar^2}\right)^{3/2}(E_v-E)^{1/2}\cdot e^{-(\mu-E)/k_BT}\,dE \\
&= \frac{1}{4}\left(\frac{2m_h^*k_BT}{\pi\hbar^2}\right)^{3/2} e^{-(\mu-E_v)/k_BT} \\
&= P_v(T)e^{-(\mu-E_v)/k_BT}
\end{aligned}
$$

$$
P_v(T)=\frac{1}{4}\left(\frac{2m_h^*k_BT}{\pi\hbar^2}\right)^{3/2} = 2.5\left(\frac{m_h^*}{m_0}\right)^{3/2}\left(\frac{T}{300\ \mathrm{K}}\right)^{3/2}\times 10^{19}\,\mathrm{cm}^{-3}
$$

Density of Carriers

$$N_c(T) = \frac{1}{4}\left(\frac{2m_e^* k_B T}{\pi\hbar^2}\right)^{3/2} = 2.5\left(\frac{m_e^*}{m_0}\right)^{3/2}\left(\frac{T}{300\ \mathrm{K}}\right)^{3/2} \times 10^{19}\ \mathrm{cm}^{-3}$$

$$P_v(T) = \frac{1}{4}\left(\frac{2m_h^* k_B T}{\pi\hbar^2}\right)^{3/2} = 2.5\left(\frac{m_h^*}{m_0}\right)^{3/2}\left(\frac{T}{300\ \mathrm{K}}\right)^{3/2} \times 10^{19}\ \mathrm{cm}^{-3}$$

effective density of states (有效态密度)

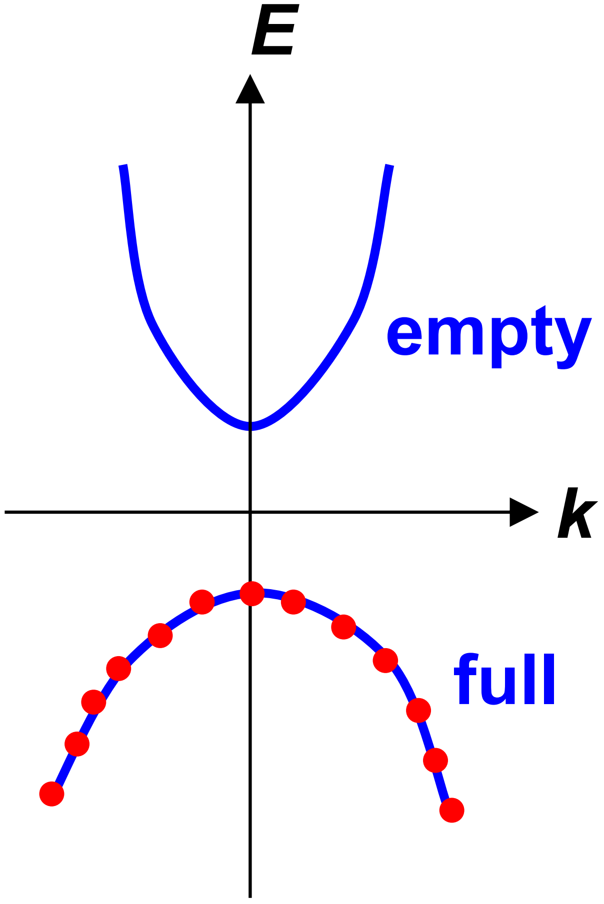
no physical meaning, just two constants

Density of Carriers

when *T* = 0 K

$$n_c = N_c(T)e^{-(E_c-\mu)/k_BT} = 0$$

$$p_v = P_v(T)e^{-(\mu-E_v)/k_BT} = 0$$



T = 0 K

insulator

Density of Carriers

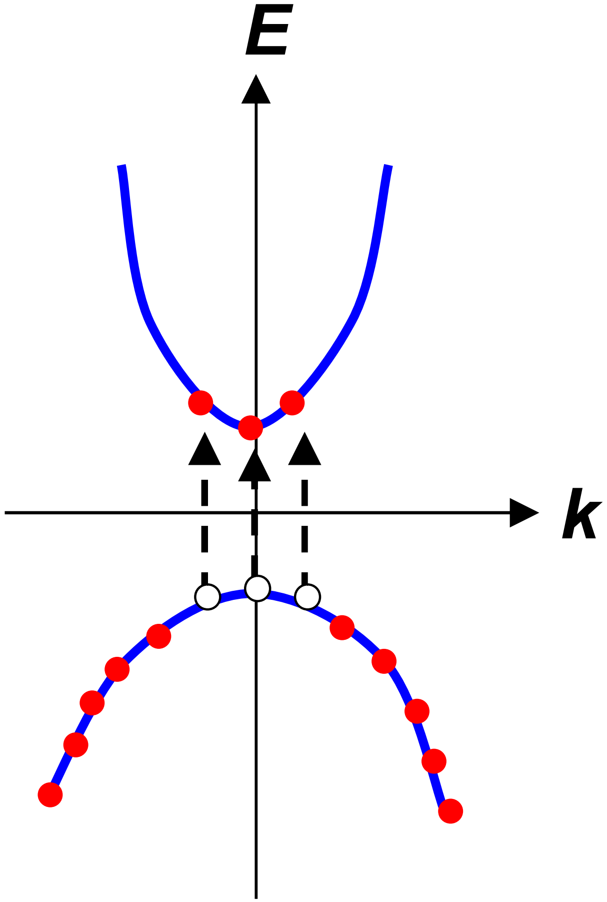
when *T* > 0 K

$$n_c = N_c(T)e^{-(E_c-\mu)/k_BT} > 0$$

$$p_v = P_v(T)e^{-(\mu-E_v)/k_BT} > 0$$

conductivity

$$\sigma = n_c e\mu_e + p_v e\mu_h$$



***T* > 0 K**

thermalization 热激发

CB and VB are partly filled

conductor

Density of Carriers

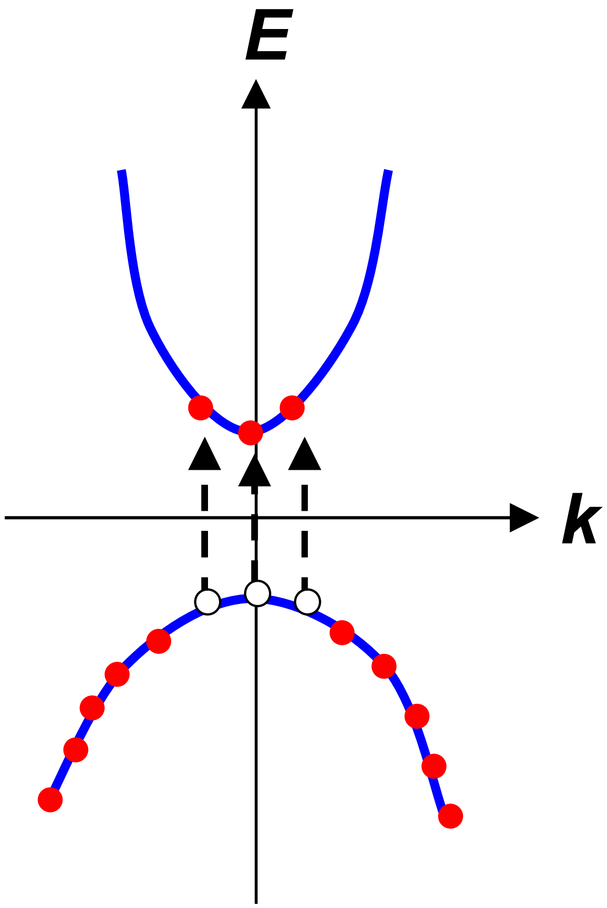
when *T* > 0 K

$$n_c = N_c(T)e^{-(E_c-\mu)/k_BT} > 0$$

$$p_v = P_v(T)e^{-(\mu-E_v)/k_BT} > 0$$

→

$$\boxed{\begin{aligned} n_c p_v &= N_v(T)P_v(T)e^{-(E_c-E_v)/k_BT} \\ &= N_v(T)P_v(T)e^{-E_g/k_BT} \end{aligned}}$$



mass action law

at equilibrium, $n_c p_v$ is a constant

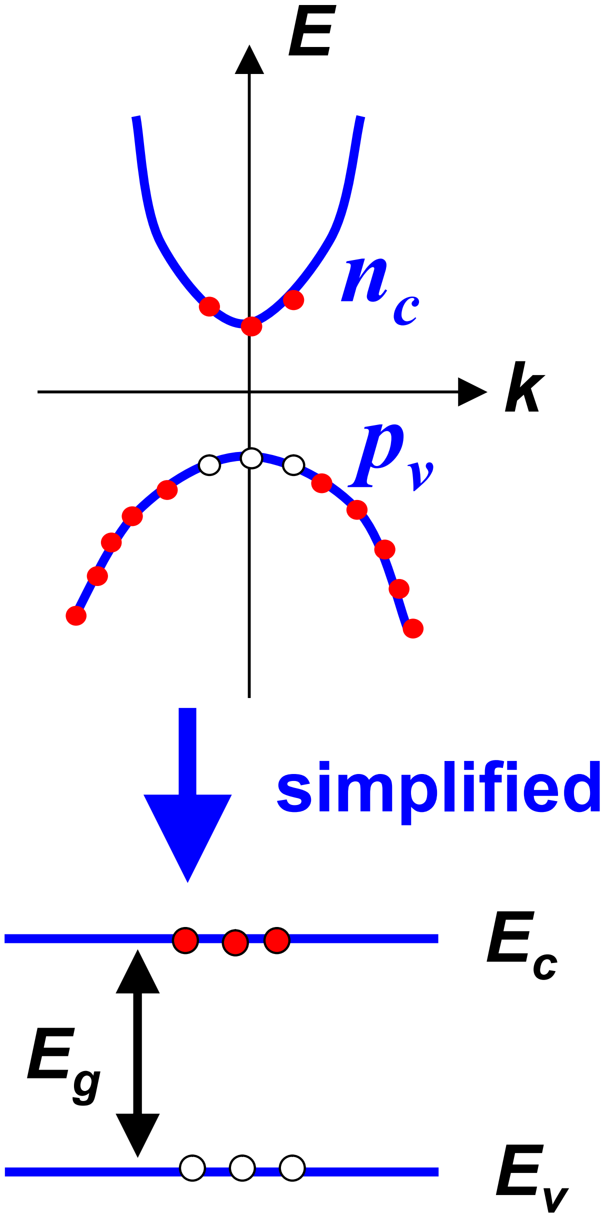
Carriers in Semiconductors

- **For calculations here, we go back to classical physics, assume:**
  - **Carriers are much fewer than DOS**

$$\boxed{f(E) \ll 1}$$

  - **Carriers are non-Degenerate (Boltzmann Distribution)**
  - **Carriers are almost in the same energies (E_c and E_v)**
  - **Carriers have the same velocities and motilities**

$$\boxed{\sigma = n_c e \mu_e + p_v e \mu_h}$$



Mass Action Law - A Little Notion

- **The product of electron and hole concentrations is a constant, at a fixed temperature**

$$\boxed{n_c p_v = n_i^2 = N_v(T) P_v(T) e^{-E_g/k_B T}}$$

- **In water, the product of H$^+$ and OH$^-$ concentrations is also a constant**

$$\boxed{[\mathrm{H}^+][\mathrm{OH}^-] = K_w = 10^{-14}\,(\mathrm{mol/L})^2 \ (\text{at } 25\ ^\circ\mathrm{C})}$$

- **Both are originated from classical statistics (non-degenerate, Maxwell-Boltzmann distribution), *not* related to quantum mechanics**

Thank you for your attention